From last time

<u>1st law</u>: ***Law of inertia***

Every object continues in its state of rest, or uniform motion in a straight line, unless acted upon by a force.

<u>2nd law</u>: ***F=ma*, or *a=F/m***

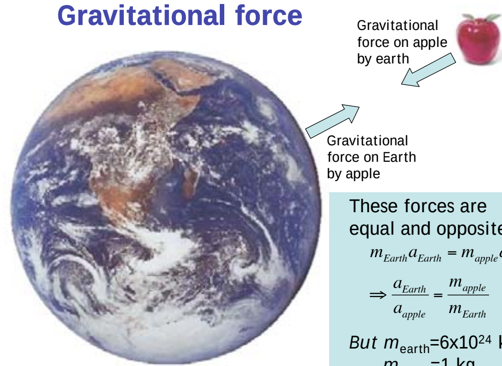
The acceleration of a body along a direction is

- proportional to the total force along that direction, and
- inversely the mass of the body

<u>3rd law</u>: ***Action and reaction***

For every action there is an equal and opposite reaction.

Gravitational force



These forces are equal and opposite,

$$m_{Earth}a_{Earth} = m_{apple}a_{apple}$$

$$\Rightarrow \frac{a_{Earth}}{a_{apple}} = \frac{m_{apple}}{m_{Earth}}$$

But m_{earth}=6x10^{24} kg
m_{apple}=1 kg

Equal accelerations

- If more massive bodies accelerate more slowly with the same force…

… why do *all* bodies fall the same, independent of mass?

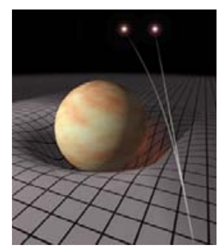
$$F_{gravity} = mg$$

- Gravitational force on a body depends on its mass:
- Therefore acceleration is independent of mass:

$$a = \frac{F_{gravity}}{m} = \frac{mg}{m} = g$$

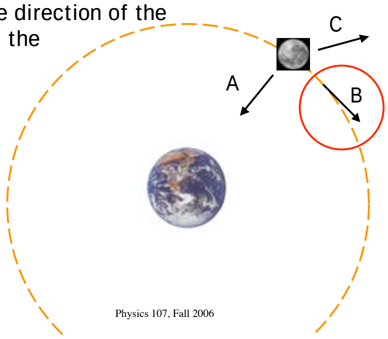
A fortunate coincidence

- A force exactly proportional to mass, so that everything cancels nicely.
- But a bit unusual.
- Einstein threw out the gravitational force entirely, attributing the observed acceleration to a distortion of space-time.



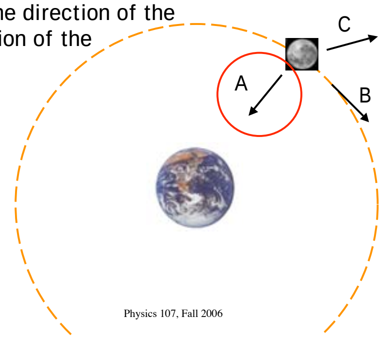
Velocity of the moon

What is the direction of the velocity of the moon?

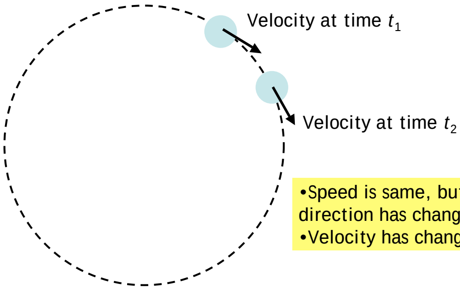


Acceleration of the moon

What is the direction of the acceleration of the moon?

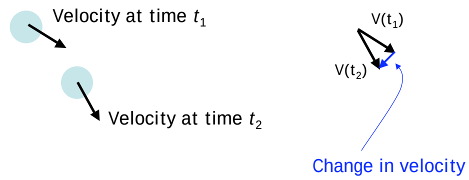


Acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$



•Speed is same, but direction has changed
•Velocity has changed

How has the velocity changed?



Centripetal acceleration = v^2/r, directed toward center of orbit. r = radius of orbit

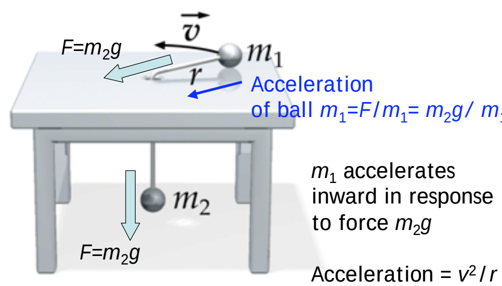
(In this equation, v is the speed of the object, which is the same at all times)

Earth's pull on the moon

- The moon continually accelerates toward the earth,
- But because of its orbital velocity, it continually **misses** the Earth.
- The orbital speed of the moon is constant, but the *direction* continually changes.
- Therefore the velocity changes with time.

True for any body in circular motion

Experiment

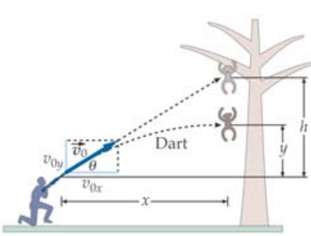


m_1 accelerates inward in response to force m_2g

Acceleration = v^2/r for circular motion

Shoot the monkey

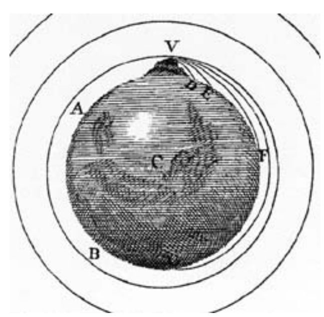
— another example of superposition



- The dart gun is fired just as the monkey drops from the tree.
- After the dart leaves the gun, the only force is from gravity.
- The only deviation from straight-line motion is an acceleration directly downward.

The monkey has exactly the same acceleration downward, so that the dart hits the monkey.

Newton's falling moon



Throwing the ball fast enough results in orbital motion

From Newton's Principia, 1615

Acceleration of moon

- The moon *is* accelerating at $\frac{v^2}{r}\ m/s^2$

  directly toward the earth!

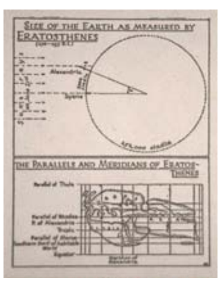
- This acceleration is due to the Earth's gravity.

- Is this acceleration different than *g*, the gravitational acceleration of an object at the Earth's surface?
  - Can calculate the acceleration directly from moon's orbital speed, and the Earth-moon distance.

Distance and diam. of moon

- The diameter of the moon is the diameter of its shadow during a solar eclipse. From the diameter d and angular size d/r~5 deg, infer distance r~60*r(earth).

The radius of the earth



- "Originally" from study of shadows at different latitudes by Eratosthenes!
- R(earth)=6500 km

Moon acceleration, cont

- Distance to moon = 60 earth radii ~ 3.84×10^8 m
- Speed of moon?

  Circumference of circular orbit = $2\pi r$

  $$\text{Speed} = \frac{\text{orbital distance} = 2\pi r}{\text{orbital time} = 27.3 \text{ days}} = 1023\ m/s$$

  Centripetal acceleration = 0.00272 m/s^2

This is the acceleration of the moon due to the gravitational force of the Earth.

Distance dependence of Gravity

- The gravitational force depends on distance.
- Moon acceleration is

  $$\frac{9.81\ \text{m/s}^2}{0.00272\ \text{m/s}^2} \approx 3600$$ times smaller than the acceleration of gravity on the Earth's surface.
- The moon is 60 times farther away, and $3600=60^2$
- So then the gravitational force drops as the distance squared

Newton: *I thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly.*

Equation for force of gravity

$$F_{gravity} \propto \frac{(\text{Mass of object 1}) \times (\text{Mass of object 2})}{\text{square of distance between them}}$$

$$F \propto \frac{m_1 \times m_2}{d^2}$$

For masses in *kilograms*, and distance in *meters*,

$$F = 6.7 \times 10^{-11} \frac{m_1 \times m_2}{d^2}$$

Example

- Find the acceleration of an apple at the surface of the earth

Force on apple = $F_{apple} = 6.7 \times 10^{-11} \frac{m_{Earth} \times m_{apple}}{d^2}$

This is also the force on the Earth by the apple!

d = distance between center of objects ~ radius of Earth

Acceleration of apple = $\frac{F_{apple}}{m_{apple}} = 6.7 \times 10^{-11} \frac{m_{Earth}}{d^2}$

$$= 6.67 \times 10^{-11} N - m^2/kg^2 \times \frac{5.98 \times 10^{24} kg}{\left(6.37 \times 10^6 m\right)^2} = 9.83 \ m/s^2$$

Gravitational force decreases with distance from Earth

Force on apple = $F_{apple} = 6.7 \times 10^{-11} \frac{m_{Earth} \times m_{apple}}{d^2}$

So moving farther from the Earth should reduce the force of gravity

- Typical airplane cruises at ~5 mi = 8000 m
  - d increases from 6,370,000 m to 6,378,000 m
  - only about a 0.25% change!

- International space station orbits at 350 km = 350,000 m
- d = 6,370,000 m + 350,000 m = 6,720,000 m
- Again d has changed only a little, so that g is decreased by only about 10%.

So why is everyone floating around?

Edward M. (Mike) Fincke, Expedition science officer

The space station is falling…

…similar to Newton's apple

- In its circular orbit, once around the Earth every 90 minutes, it is continuously accelerating toward the Earth at ~8.8 m/s^2.
- Everything inside it is also falling (accelerating toward Earth at that same rate).
- The astronauts are freely falling inside a freely-falling 'elevator'. They have the perception of weightlessness, since their environment is falling just as they are.

Supreme Scream - 300 feet of pure adrenaline rush

A freefall ride

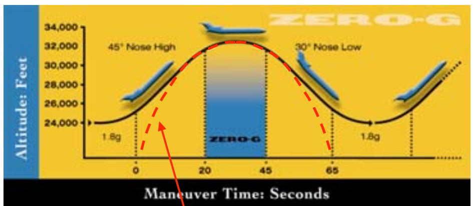
$$d = \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2d}{a}}$$

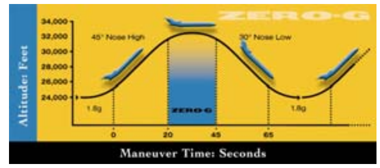
$$= \sqrt{\frac{2 \times 300 \, ft}{32 \ \ ft/s^2}}$$

$= 4.3$ sec of freefall

A little longer ride



Parabolic path of freely falling object



Acceleration of gravity on moon

- On the moon, an apple feels gravitational force from the moon.
- Earth is too far away.

Force on apple on moon $= F_{apple} = 6.7\times10^{-11}\dfrac{m_{moon}\times m_{apple}}{r_{moon}^2}$

Accel. of apple on moon $= \dfrac{F_{apple}}{m_{apple}} = 6.7\times10^{-11}\dfrac{m_{moon}}{r_{moon}^2}$

Compare to accel on Earth $= 6.7\times10^{-11}\dfrac{m_{Earth}}{r_{Earth}^2}$

$$\frac{\text{accel. on moon}}{\text{accel. on Earth}} = \frac{m_{moon}/m_{Earth}}{\left(r_{moon}/r_{Earth}\right)^2}$$

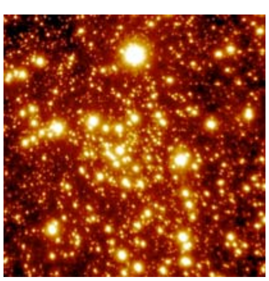
Accel. of gravity on moon

$$\frac{\text{accel. on moon}}{\text{accel. on Earth}} = \frac{m_{moon}/m_{Earth}}{\left(r_{moon}/r_{Earth}\right)^2}$$

$$= \frac{7.4\times10^{22}\,kg/6.0\times10^{24}\,kg}{\left(1.7\times10^{6}\,m/6.4\times10^{6}\,m\right)^2}$$

$$= \frac{0.0123}{(.265)^2} = 0.175 \approx \frac{1}{6}$$

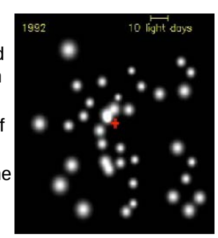
Gravitational force at large distances: Stars orbiting our black hole



- At the center of our galaxy is a collection of stars found to be in motion about an invisible object.

Orbits obey Newton's gravity, orbiting around some central mass

- Scientists at the Max Planck Institute for Extraterrestrische Physik has used infrared imaging to study star motion in the central parsec of our galaxy.
- Movie at right summarizes 14 years of observations.
- Stars are in orbital motion about some massive central object



http://www.mpe.mpg.de/www_ir/GC/intro.html

What is the central mass?

- One star swings by the hole at a minimum distance b of 17 light hours (120 A.U. or close to three times the distance to Pluto) at speed v=5000 km/s, period 15 years.
- From the orbit we can derive the mass.
- The mass is 2.6 million solar masses.
- It is mostly likely a black hole at the center of our Milky Way galaxy!