Homework - Exam

HW#6:
Chap 10 Conceptual: 36, 42 Problem 7, 9
Chap 11 Conceptual: 5, 10

Hour Exam 2: Wednesday, October 25th

- In-class, covering waves, electromagnetism, and relativity
- Twenty multiple-choice questions
- Will cover: Chapters 8, 9 10 and 11
  Lecture material
- You should bring
  - 1 page notes, written single sided
  - #2 Pencil and a Calculator
  - Review Monday October 23rd
  - Review test online on Monday

From last time...

- Einstein's Relativity
  - **All laws of physics identical in inertial ref. frames**
  - **Speed of light=*c* in all inertial ref. frames**
- Consequences
  - *Simultaneity:* **events simultaneous in one frame will not be simultaneous in another.**
  - *Time dilation*
  - *Length contraction*
  - *Relativistic invariant:* x^2-c^2t^2 **is 'universal' in that it is measured to be the same for all observers**

Review: Time Dilation and Length Contraction

Time in other frame

Time in object's rest frame

$$\boxed{T = \gamma T_p = T_p \Big/ \sqrt{1 - v^2/c^2}}$$

Times measured in other frames are longer (time dilation)

Length in other frame

Length in object's rest frame

$$\boxed{L = \frac{L_p}{\gamma} = L_p\sqrt{1 - \frac{v^2}{c^2}}}$$

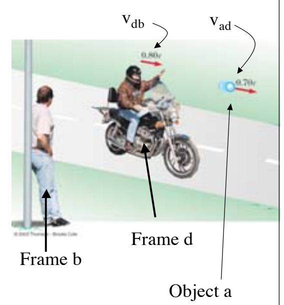
Distances measured in other frames are shorter (length contraction)

- Need to define the rest frame and the "other" frame which is moving with respect to the rest frame

Relativistic Addition of Velocities

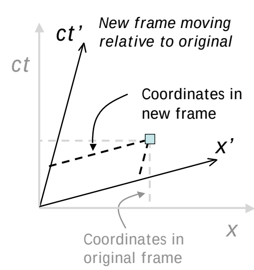
- As motorcycle velocity approaches *c*, v_{ab} also gets closer and closer to *c*
- End result: nothing exceeds the speed of light

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}}$$



Observing from a new frame

- In relativity these events will look different in reference frame moving at some velocity
- The new reference frame can be represented as same events along different coordinate axes



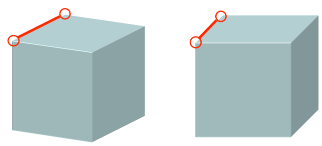
A relativistic invariant quantity

Earth Frame	Ship Frame
Event separation = 4.3 LY	Event separation = 0 LY
Time interval = 4.526 yrs	Time interval = 1.413 yrs
$(\text{separation})^2 - c^2(\text{time interval})^2$ $= (4.3)^2 - (c(4.526\,yrs))^2 = -2.0\ LY^2$	$(\text{separation})^2 - c^2(\text{time interval})^2$ $= 0 - (c(1.413\,yrs))^2 = -2.0\ LY^2$

- The quantity (separation)^2-c^2(time interval)^2 is the same for all observers
- It mixes the space and time coordinates

‘Separation’ between events

- Views of the same cube from two different angles.
- Distance between corners (length of red line drawn on the flat page) seems to be different depending on how we look at it.



- But clearly this is just because we are not considering the full three-dimensional distance between the points.
- The 3D distance does not change with viewpoint.

Newton again

- Fundamental relations of Newtonian physics
  - *acceleration = (change in velocity)/(change in time)*
  - *acceleration = Force / mass*
  - *Work = Force x distance*
  - *Kinetic Energy = (1/2) (mass) x (velocity)^2*
  - *Change in Kinetic Energy = net work done*
- Newton predicts that a constant force gives
  - Constant acceleration
  - Velocity proportional to time
  - Kinetic energy proportional to (velocity)^2

Forces, Work, and Energy in Relativity
What about Newton’s laws?

- Relativity dramatically altered our perspective of space and time
  - But clearly objects still move,
    spaceships are accelerated by thrust,
    work is done,
    energy is converted.
- How do these things work in relativity?

Applying a constant force

- Particle initially at rest,
  then subject to a **constant force** starting at *t*=0,

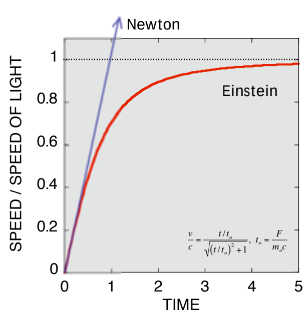
  Δmomentum =momentum = (*Force*) x (*time*)

- Using momentum = (mass) x (velocity),
  Velocity increases without bound as time increases

Relativity says ***no***.
The effect of the force gets smaller and smaller as velocity approaches speed of light

Relativistic speed of particle subject to constant force

- At small velocities (short times) the motion is described by Newtonian physics
- At higher velocities, big deviations!
- The velocity never exceeds the speed of light



Momentum in Relativity

- The relationship between momentum and force is very simple and fundamental

Momentum is constant for zero force

and

$$\frac{\text{change in momentum}}{\text{change in time}} = \text{Force}$$

This relationship is preserved in relativity

Relativistic momentum

- Relativity concludes that the Newtonian definition of momentum (p_{Newton}=mv=mass x velocity) is accurate at low velocities, but not at high velocities

The **relativistic momentum** is:

Relativistic gamma

mass

$$p_{relativistic} = \gamma m v$$

velocity

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Was Newton wrong?

- Relativity requires a different concept of momentum

$$p_{relativistic} = \gamma m v$$
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

- But not really so different!
- For small velocities << light speed $\gamma \approx 1$, and so $p_{relativistic} \approx mv$
- This is Newton's momentum
- Differences only occur at velocities that are a substantial fraction of the speed of light

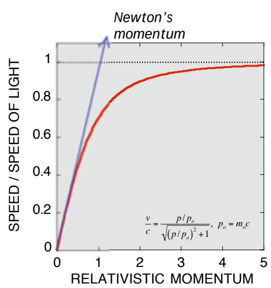
Relativistic Momentum

- Momentum can be increased arbitrarily, but velocity never exceeds c
- We still use

$$\frac{\text{change in momentum}}{\text{change in time}} = \text{Force}$$

- For constant force we still have momentum = Force x time, but the velocity never exceeds c
- Momentum has been redefined

$$p_{relativistic} = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}}$$



Relativistic momentum for different speeds.

How can we understand this?

- acceleration $\left(=\frac{\text{change in velocity}}{\text{change in time}}\right)$ much smaller at high speeds than at low speeds
- Newton said force and acceleration related by mass.
- We could say that mass increases as speed increases.

$$p_{relativistic} = \gamma m v = (\gamma m) v \equiv m_{relativistic} v$$

- Can write this

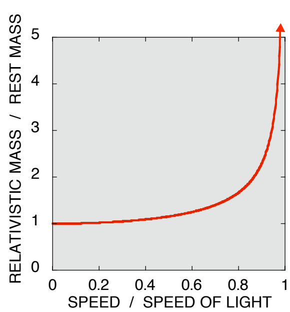
$$p_{relativistic} = \gamma m_o v = (\gamma m_o) v \equiv mv$$
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}, \quad m = \gamma m_o$$

— m_o is the ***rest mass***.

— relativistic mass m depends on velocity

Relativistic mass

- The the particle becomes extremely massive as speed increases ($m=\gamma m_o$)
- The relativistic momentum has new form ($p= \gamma m_o v$)
- Useful way of thinking of things remembering the concept of inertia



Example

- An object moving at half the speed of light relative to a particular observer has a rest mass of 1 kg. What is it's mass measured by the observer?

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.5c/c)^2}} = \frac{1}{\sqrt{1-0.25}}$$
$$= \frac{1}{\sqrt{0.75}} = 1.15$$

So measured mass is 1.15kg

Question

A object of rest mass of 1 kg is moving at 99.5% of the speed of light.
What is it's measured mass?

A. 10 kg
B. 1.5 kg
C. 0.1 kg

Relativistic Kinetic Energy

- Might expect this to change in relativity.
- Can do the same analysis as we did with Newtonian motion to find

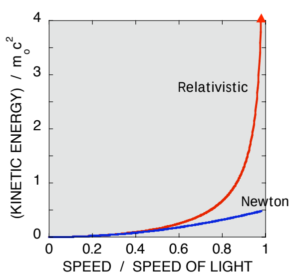
$$KE_{relativistic} = (\gamma - 1)m_o c^2$$

- Doesn't seem to resemble Newton's result at all
- However for small velocities, it does reduce to the Newtonian form

$$KE_{relativistic} \approx \frac{1}{2} m_o v^2 \quad for \;\; v << c$$

Relativistic Kinetic Energy

- Can see this graphically as with the other relativistic quantities
- Kinetic energy gets arbitrarily large as speed approaches speed of light
- Is the same as Newtonian kinetic energy for small speeds.



Total Relativistic Energy

- The relativistic kinetic energy is

$$KE_{relativistic} = (\gamma - 1)m_o c^2$$
$$= \underbrace{\gamma m_o c^2}_{\text{Depends on velocity}} - \underbrace{m_o c^2}_{\text{Constant, independent of velocity}}$$

- Write this as

$$\underbrace{\gamma m_o c^2}_{\text{Total energy}} = \underbrace{KE_{relativistic}}_{\text{Kinetic energy}} + \underbrace{m_o c^2}_{\text{Rest energy}}$$

Mass-energy equivalence

- This results in Einstein's famous relation

$$E = \gamma m_o c^2, \quad or \quad E = mc^2$$

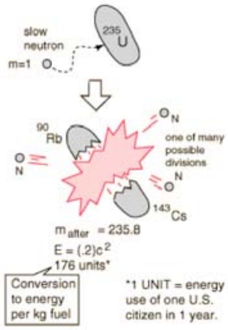
- This says that the total energy of a particle is related to its mass.
- Even when the particle is not moving it has energy.
- We could also say that mass is another form of energy
  - Just as we talk of chemical energy, gravitational energy, etc, we can talk of **mass energy**

Example

- In a frame where the particle is at rest, its total energy is $E = m_o c^2$
- Just as we can convert electrical energy to mechanical energy, it is possible to tap mass energy
- A 1 kg mass has (1kg)(3x10^{8}m/s)^2=9x10^{16} J of energy
  - We could power 30 million 100 W light bulbs for one year! (~30 million sec in 1 yr)

Nuclear Power

- Doesn't convert whole protons or neutrons to energy
- Extracts some of the binding energy of the nucleus
  - ^{90}Rb and ^{143}Cs + 3n have less rest mass than ^{235}U +1n: $E = mc^2$



Energy and momentum

- Relativistic energy is $E = \gamma m_o c^2$
- Since γ depends on velocity, the energy is measured to be different by different observers
- Momentum also different for different observers
  - Can think of these as analogous to space and time, which individually are measured to be different by different observers
- But there is something that is the same for all observers:

$$E^2 - c^2 p^2 = \left(m_o c^2\right)^2$$ = Square of rest energy

- Compare this to our space-time invariant $x^2 - c^2 t^2$

A relativistic perspective

- The concepts of space, time, momentum, energy that were useful to us at low speeds for Newtonian dynamics are a little confusing near light speed
- Relativity needs new conceptual quantities, such as space-time and energy-momentum
- Trying to make sense of relativity using space and time separately leads to effects such as time dilation and length contraction
- In the mathematical treatment of relativity, space-time and energy-momentum objects are always considered together

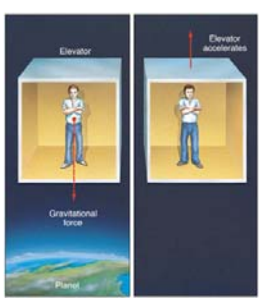
The Equivalence Principle

Clip from Einstein Nova special

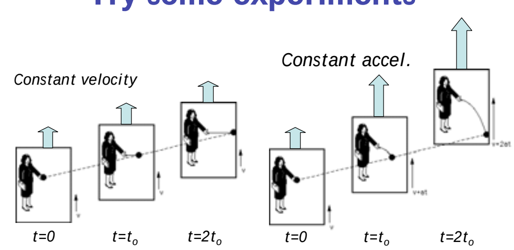
- Led Einstein to postulate the **Equivalence Principle**

Equivalence principle

Accelerating reference frames are **indistinguishable** from a gravitational force



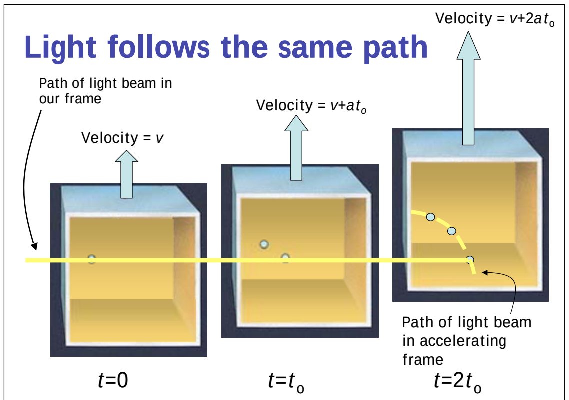
Try some experiments



Floor accelerates upward to meet ball

Cannot do any experiment to distinguish accelerating frame from gravitational field

Light follows the same path



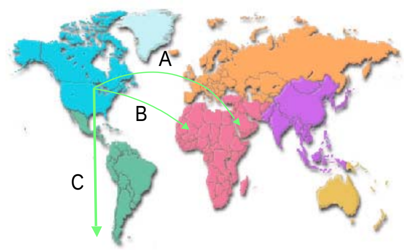
Is light bent by gravity?

- If we can't distinguish an accelerating reference frame from gravity…
- and light bends in an accelerating reference frame…
- then light must bend in a gravitational field

But light doesn't have any mass.
How can gravity affect light?

Maybe we are confused about what a straight line is

Which of these is a straight line?



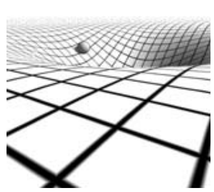
A. A
B. B
C. C
D. All of them

Straight is shortest distance

- They are the shortest distances determined by wrapping string around a globe. On a globe, they are called 'great circles'. In general, geodesics.
- This can be a general definition of straight, and is in fact an intuitive one on curved surfaces
- It is the one Einstein used for the path of all objects in curved space-time
- The confusion comes in when you don't know you are on a curved surface.

Mass and curvature

- General relativity says that any mass will give space-time a curvature
- Motion of objects in space-time is determined by that curvature
- Similar distortions to those we saw when we tried to draw graphs in special relativity



Idea behind geometric theory

- Matter bends space and time.
- Bending on a two-dimensional surface is characterized by curvature at each point

curvature = 1/(radius of curvature)^2

- How can we relate curvature to matter?

Einstein's solution

- Einstein guessed that the curvature functions
  (units of $1/m^2$)
  are proportional to
  the local energy and momentum densities
  (units of kg/m^3)
- The proportionality constant from comparison with Newtonian theory is

$$\frac{8\pi G}{c^2}$$

where G is Newton's constant

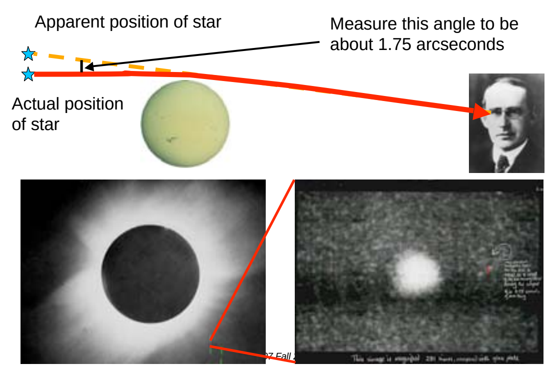
Near the Earth

- The ratio of the curvature of space on the surface of the Earth to the curvature of the surface of the Earth is
  ~ 7×10^{-10}
- The curvature of space near Earth is so small as to be usually unnoticeable.
- But is does make objects accelerate toward the earth!

A test of General Relativity

- Can test to see if the path of light appears curved to us
- Local massive object is the sun
- Can observe apparent position of stars with and without the sun
- But need to block glare from sun

Eddington and the Total Eclipse of 1919



Eddington's Eclipse Expedition 1919

- Eddington, British astronomer, went to Principe Island in the Gulf of Guinea to observe solar eclipse.
- After months of drought, it was pouring rain on the day of the eclipse
- Clouds parted just in time, they took photographic plates showing the location of stars near the sun.
- Analysis of the photographs back in the UK produced a deflection in agreement with the GR prediction