From Last Time…

- Light waves are particles
  and matter particles are waves!
- Electromagnetic radiation (e.g. light) made up of photon particles
- Matter particles show wavelike properties like interference

HW #7: Chapter 13: Conceptual: # 8, 11, 25, 27
Problems: # 4, 12

Due: Nov 8th

Essay: Topic and paragraph due Nov 3rd

Photon: particle and wave

- *Light: Is quantized. Has energy and momentum:*

$$E = hf = \frac{hc}{\lambda} = \frac{1240\ eV-nm}{\lambda} \qquad p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \lambda = \frac{h}{p}$$

- *Electromagnetic radiation(light) has a dual nature. It exhibits both wave and particle characteristics*
- The photoelectric effect
  shows the particle characteristics of light
- Interference and diffraction
  shows the wave and particle properties and the probabilistic aspect of quantum mechanics

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Wavelengths of massive objects

- deBroglie wavelength = $\lambda = \frac{h}{p}$
- *p*=*mv* for a nonrelativistic (*v*<<*c*) particle **with mass**.

$$\lambda = \frac{h}{mv}$$

Same constant as before

$$\lambda = \frac{h}{p} = \frac{\boxed{hc}}{\sqrt{2\boxed{mc^2}\boxed{E_{kinetic}}}}$$

kinetic energy

rest energy

Wavelength of eV electrons

- For an electron,

constant

$$\lambda = \frac{\boxed{1240\ \ eV-nm}}{\sqrt{2\times\boxed{0.511\ \ MeV}}}\frac{1}{\sqrt{\boxed{E_{kinetic}}}} = \frac{1.23\ \ eV^{1/2}-nm}{\sqrt{E_{kinetic}}}$$

rest energy *kinetic energy*

- 1 eV electron, λ=1.23 nm
- 10 eV electron λ=0.39 nm
- 100 eV electron λ=0.12 nm

Wavelength of 100 eV objects

- For an electron,

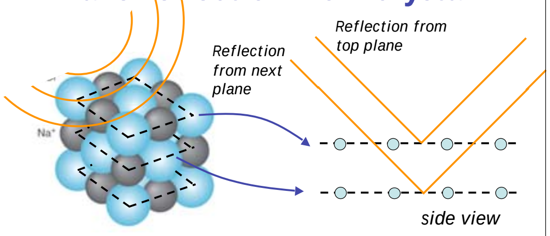
constant

$$\lambda = \frac{\boxed{1240\ \ eV-nm}}{\sqrt{2\times\boxed{m_0 MeV}}}\frac{1}{\sqrt{\boxed{100eV}}} = \frac{88\ \ eV^{1/2}-nm}{\sqrt{m_0 MeV}}$$

rest energy *kinetic energy*

- 100 eV electron, λ=0.12 nm
- 100 eV proton λ=0.0029 nm = 2.9 pm
- Electron .511 MeV, Proton 940 MeV

Wave reflection from crystal

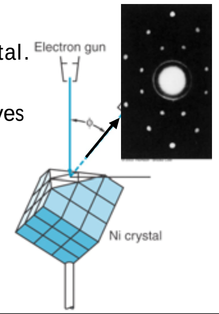


- If electron are waves they can interfere
- Interference of waves reflecting from different atomic layers in the crystal.
- Difference in path length ~ spacing between atoms

Davisson-Germer experiment

- Diffraction of electrons from a nickel single crystal.
- Established that electrons are waves

Bright spot: constructive interference



54 eV electrons (λ=0.17nm)

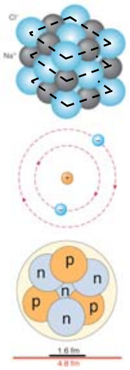
Davisson: Nobel Prize 1937

Particle interference

- Used this interference idea to to learn about the structure of matter

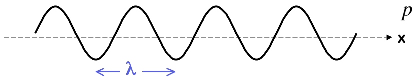
$$\lambda = \frac{1240\ \ eV - nm}{\sqrt{2 \times m_0 MeV}} \frac{1}{\sqrt{KE}}$$

- 100 eV electrons: λ = 0.12nm
  - Crystals also the atom
- 10 GeV electrons:
  - Inside the nucleus, 3.2 fermi, 10^{-6} nm
- 10 GeV protons:
  - Inside the protons and neutrons: .29 fermi



Let's study electron waves

- Here is a wave:



$$\lambda = \frac{h}{p}$$

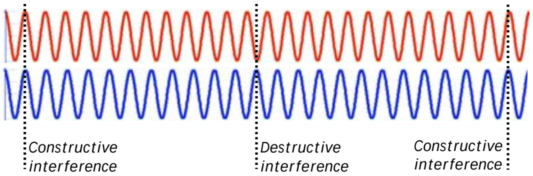
...where is the electron?
  - Wave extends infinitely far in +*x* and -*x* direction

Analogy with sound

- Sound wave also has the same characteristics
- But we can often locate sound waves
  - *E.g.* echoes bounce from walls. Can make a sound pulse
- Example:
  - Hand clap: duration ~ 0.01 seconds
  - Speed of sound = 340 m/s
  - Spatial extent of sound pulse = 3.4 meters.
  - 3.4 meter long hand clap travels past you at 340 m/s

Beat frequency: spatial localization

- What does a sound 'particle' look like?
  - One example is a 'beat frequency' between two notes
  - Two sound waves of almost same wavelength added.

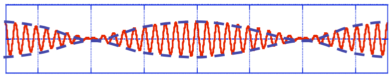


Large amplitude

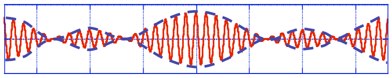
Small amplitude

Large amplitude

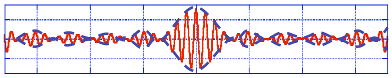
Making a particle out of waves



440 Hz +
439 Hz

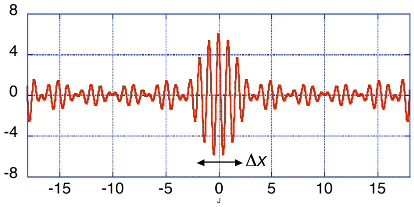


440 Hz +
439 Hz +
438 Hz



440 Hz +
439 Hz +
438 Hz +
437 Hz +
436 Hz

Spatial extent of localized sound wave

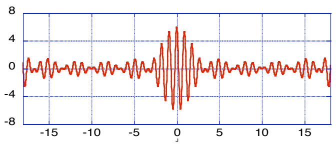


- Δx = spatial spread of 'wave packet'
- Spatial extent decreases as the spread in included wavelengths increases.

Same occurs for a matter wave

- Construct a localized particle by adding together waves with slightly different wavelengths.
- Since de Broglie says $\lambda = h\ /p$, each of these components has slightly different momentum.
  - We say that there is some 'uncertainty' in the momentum or the energy
- And still don't know exact location of the particle!
  - Wave still is spread over Δx ('uncertainty' in position)
  - Can reduce Δx, but at the cost of increasing the spread in wavelength (giving a spread in momentum).

Interpreting



- For sound, we would just say that the sound pulse is centered at some position, but has a spread.
- Can't do that for a quantum-mechanical particle.
- Many measurements indicate that the electron is indeed a point particle.
- Interpretation is that the magnitude of electron 'wave-pulse' at some point in space determines the **probability** of finding the electron at that point.

Heisenberg Uncertainty Principle

- Using
  - Δx = position uncertainty
  - Δp = momentum uncertainty
- Heisenberg showed that the product

Planck's constant

$(\ \Delta x\ ) \bullet (\ \Delta p\ )$ is always greater than $(\ h\ /\ 4\pi\ )$

Often write this as $(\Delta x)(\Delta p) \sim \hbar/2$

where $\hbar = \dfrac{h}{2\pi}$ is pronounced 'h-bar'

Thinking about uncertainty

$$(\Delta x)(\Delta p) \sim \hbar/2$$

For a classical particle, $p=mv$, so an uncertainty in momentum corresponds to an uncertainty in velocity.

$$(\Delta x)(\Delta v) \sim \hbar/2m$$

This says that the uncertainty is small for massive objects, but becomes important for very light objects, such as electrons.

Large, massive objects don't show effects of quantum mechanics.

Uncertainty principle question

Suppose an electron is inside a box 1 nm in width. There is some uncertainty in the momentum of the electron. We then squeeze the box to make it 0.5 nm. What happens to the momentum?

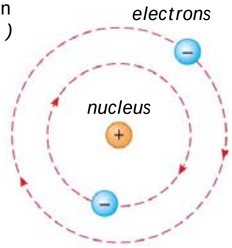
[✓ A. Momentum becomes more uncertain]
B. Momentum becomes less uncertain
C. Momentum uncertainty unchanged

Using quantum mechanics

- Quantum mechanics makes astonishingly accurate predictions of the physical world
- Can apply to atoms, molecules, solids.
- An early success was in understanding
  - Structure of atoms
  - Interaction of electromagnetic radiation with atoms

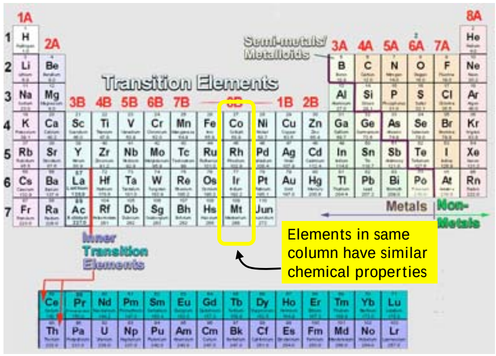
Planetary model of atom

- Positive charge is concentrated in the center of the atom (*nucleus*)
- Atom has zero net charge:
  - Positive charge in nucleus cancels negative electron charges.
- Electrons orbit the nucleus like planets orbit the sun
- (Attractive) Coulomb force plays role of gravity



Difference between atoms

- No net charge to atom
  - number of orbiting negative electrons same as number of positive protons in nucleus
  - Different elements have different number of orbiting electrons
- Hydrogen: 1 electron
- Helium: 2 electrons
- Copper: 29 electrons
- Uranium: 92 electrons!
- Organized into periodic table of elements



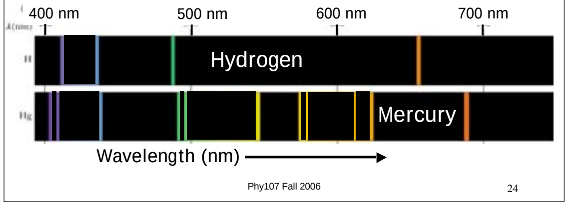
Elements in same column have similar chemical properties

Planetary model and radiation

- Circular motion of orbiting electrons causes them to emit electromagnetic radiation with frequency equal to orbital frequency.
- Same mechanism by which radio waves are emitted by electrons in a radio transmitting antenna.
- In an atom, the emitted electromagnetic wave carries away energy from the electron.
  - Electron predicted to continually lose energy.
  - The electron would eventually spiral into the nucleus
  - *However most atoms are stable!*

Atoms and photons

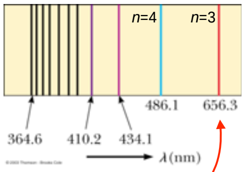
- Experimentally, atoms do emit electromagnetic radiation, but not just any radiation!
- In fact, each atom has its own 'fingerprint' of different light frequencies that it emits.



Hydrogen emission spectrum

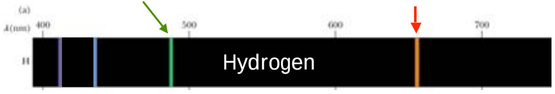
- Hydrogen is simplest atom
  - One electron orbiting around one proton.
- The **Balmer Series** of emission lines empirically given by

$$\frac{1}{\lambda_m}=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$



$n = 4$, $\lambda = 486.1$ nm

$n = 3$, $\lambda = 656.3$ nm

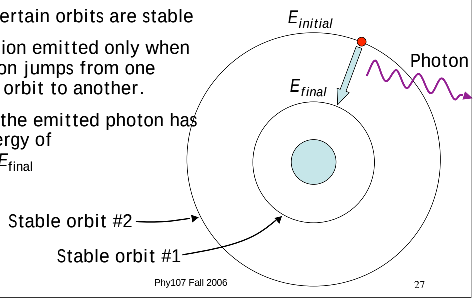


Hydrogen emission

- This says hydrogen emits only photons of a particular wavelength, frequency
- Photon energy = *hf*, so this means a particular energy.
- Conservation of energy:
  - Energy carried away by photon is lost by the orbiting electron.

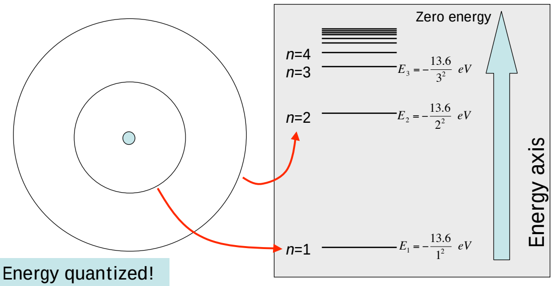
The Bohr hydrogen atom

- Retained ‘planetary’ picture: one electron orbits around one proton
- Only certain orbits are stable
- Radiation emitted only when electron jumps from one stable orbit to another.
- Here, the emitted photon has an energy of $E_{\text{initial}}-E_{\text{final}}$



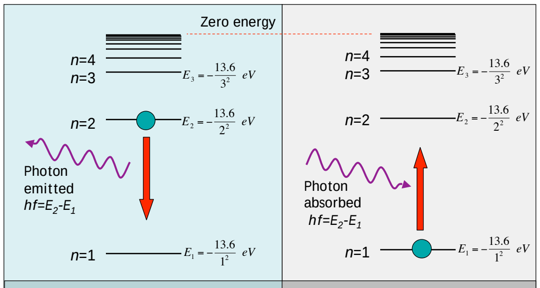
Energy levels

- Instead of drawing orbits, we can just indicate the energy an electron would have if it were in that orbit.



Energy quantized!

Emitting and absorbing light



Photon is emitted when electron drops from one quantum state to another

Absorbing a photon of correct energy makes electron jump to higher quantum state.

Photon emission question

An electron can jump between the allowed quantum states (energy levels) in a hydrogen atom. The lowest three energy levels of an electron in a hydrogen atom are -13.6 eV, -3.4 eV, -1.5 eV.

These are part of the sequence $E_n = -13.6/n^2$ eV.

Which of the following photons could be emitted by the hydrogen atom?

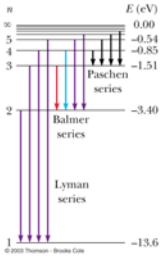
A. 10.2 eV
B. 3.4 eV
C. 1.7 eV

The energy carried away by the photon must be given up by the electron. The electron can give up energy by dropping to a lower energy state.

So possible photon energies correspond to differences between electron orbital energies. The 10.2 eV photon is emitted when the electron jumps from the -3.4 eV state to the -13.6 eV state, losing 10.2 eV of energy.

Energy conservation for Bohr atom

- Each orbit has a specific energy
  $E_n = -13.6/n^2$
- Photon emitted when electron jumps from high energy to low energy orbit.
  $E_i - E_f = h f$
- Photon absorption induces electron jump from low to high energy orbit.
  $E_f - E_i = h f$
- Agrees with experiment!



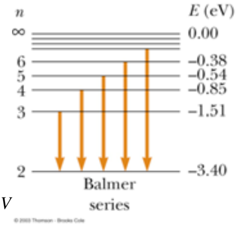
Example: the Balmer series

- All transitions terminate at the *n*=2 level
- Each energy level has energy $E_n = -13.6 \,/\, n^2$ eV
- E.g. *n*=3 to *n*=2 transition
  - Emitted photon has energy

$$E_{photon} = \left(\left(-\frac{13.6}{3^2}\right) - \left(-\frac{13.6}{2^2}\right)\right) = 1.89 \;\; eV$$

  - Emitted wavelength

$$E_{photon} = hf = \frac{hc}{\lambda}, \quad \lambda = \frac{hc}{E_{photon}} = \frac{1240 \;\; eV - nm}{1.89 \;\; eV} = 656 \;\; nm$$



Spectral Question

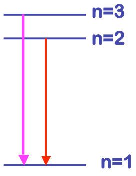
Compare the wavelength of a photon produced from a transition from n=3 to n=1 with that of a photon produced from a transition n=2 to n=1.

A. $\lambda_{31} < \lambda_{21}$

B. $\lambda_{31} = \lambda_{21}$

C. $\lambda_{31} > \lambda_{21}$

$E_{31} > E_{21}$ so $\lambda_{31} < \lambda_{21}$



But why?

- Why should only certain orbits be stable?
- Bohr had a complicated argument based on "correspondence principle"
  - That quantum mechanics must agree with classical results when appropriate (high energies, large sizes)
- But incorporating wave nature of electron gives a natural understanding of these 'quantized orbits'