From Last Time…

- Ideas of quantum mechanics
- Electromagnetic(Light) waves are particles and matter particles are waves!
- Multiple results of an experiment are possible each with it own probability
- Photons and matter particles are spread out over a small volume

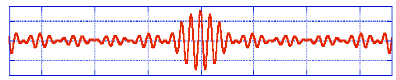
Today

- Quantum mechanics of the atom

Particle and wave

$$\lambda = \frac{h}{p}$$

- Every particle has a wavelength
- However, particles are at approximately one position.
  - Works if the particles has a superposition nearby of wavelengths rather than one definite wavelength



440 Hz +
439 Hz +
438 Hz +
437 Hz +
436 Hz

- Heisenberg uncertainty principle $(\Delta x)(\Delta p) \sim \hbar/2$
  - However particle is still spread out over small volume in addition to being spread out over several wavelengths

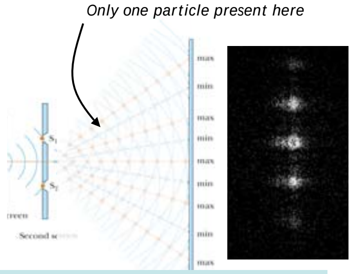
Particle interference

Do an interference experiment.

But turn down the intensity until only ONE particle at a time is between slits and screen

Is there still interference?

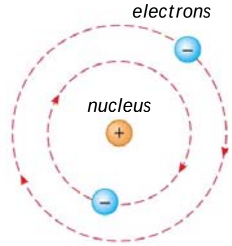
Only one particle present here



In addition to the idea of probabilities we needed the idea of the particle filling a finite volume so that it could go through both slits and interfere with itself.

Planetary model of atom

- Positive charge is concentrated in the center of the atom (*nucleus*)
- Atom has zero net charge:
  - Positive charge in nucleus cancels negative electron charges.
- Electrons orbit the nucleus like planets orbit the sun
- (Attractive) Coulomb force plays role of gravity

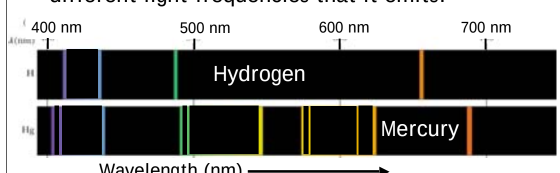


Planetary model and radiation

- Circular motion of orbiting electrons causes them to emit electromagnetic radiation with frequency equal to orbital frequency.
- Same mechanism by which radio waves are emitted by electrons in a radio transmitting antenna.
- In an atom, the emitted electromagnetic wave carries away energy from the electron.
  - Electron predicted to continually lose energy.
  - The electron would eventually spiral into the nucleus
  - *However most atoms are stable!*

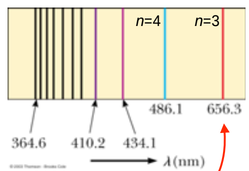
Atoms and photons

- Experimentally, atoms do emit electromagnetic radiation, but not just any radiation!
- In fact, each atom has its own 'fingerprint' of different light frequencies that it emits.



Hydrogen emission spectrum

- Hydrogen is simplest atom
  - One electron orbiting around one proton.
- The **Balmer Series** of emission lines empirically given by



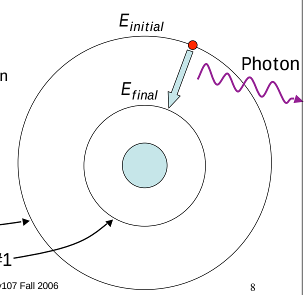
$$\frac{1}{\lambda_m} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$n = 4$, $\lambda = 486.1$ nm

$n = 3$, $\lambda = 656.3$ nm

The Bohr hydrogen atom

- Retained 'planetary' picture: one electron orbits around one proton
- Only certain orbits are stable
- Radiation emitted only when electron jumps from one stable orbit to another.
- Here, the emitted photon has an energy of $E_{initial}$-E_{final}

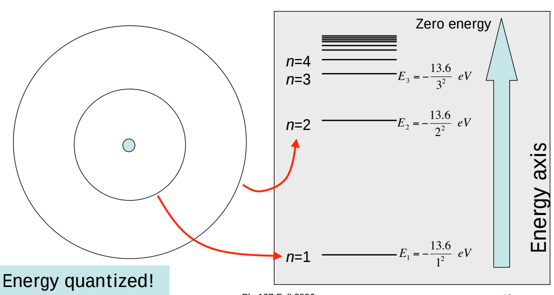


Hydrogen emission

- This says hydrogen emits only photons at particular wavelengths, frequencys
- Photon energy = hf, so this means a particular energy.
- Conservation of energy:
  - Energy carried away by photon is lost by the orbiting electron.

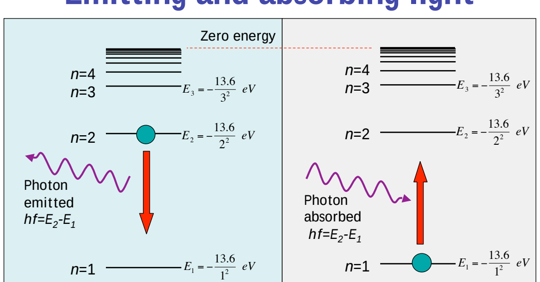
Energy levels

- Instead of drawing orbits, we can just indicate the energy an electron would have if it were in that orbit.



Energy quantized!

Emitting and absorbing light



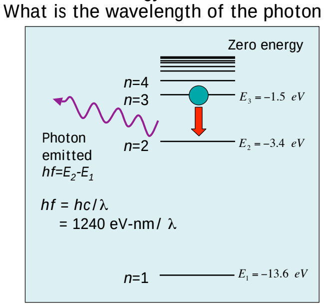
Photon is emitted when electron drops from one quantum state to another

Absorbing a photon of correct energy makes electron jump to higher quantum state.

Hydrogen atom

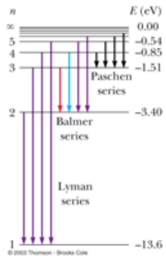
An electron drops from an -1.5 eV energy level to one with energy of -3.4 eV. What is the wavelength of the photon emitted?

A. 650 nm
B. 400 nm
C. 250 nm



Energy conservation for Bohr atom

- Each orbit has a specific energy
  $E_n = -13.6/n^2$
- Photon emitted when electron jumps from high energy to low energy orbit.
  $E_i - E_f = h f$
- Photon absorption induces electron jump from low to high energy orbit.
  $E_f - E_i = h f$
- Agrees with experiment!



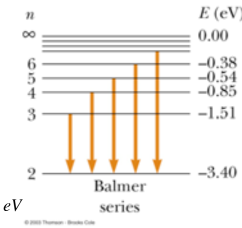
Example: the Balmer series

- All transitions terminate at the n=2 level
- Each energy level has energy E_n=-13.6 / n^2 eV
- E.g. n=3 to n=2 transition
  - Emitted photon has energy

$$E_{photon} = \left(\left(-\frac{13.6}{3^2} \right) - \left(-\frac{13.6}{2^2} \right) \right) = 1.89 \ eV$$

  - Emitted wavelength

$$E_{photon} = hf = \frac{hc}{\lambda}, \quad \lambda = \frac{hc}{E_{photon}} = \frac{1240 \ eV - nm}{1.89 \ eV} = 656 \ nm$$

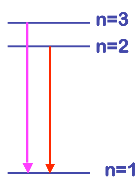


Spectral Question

Compare the wavelength of a photon produced from a transition from n=3 to n=1 with that of a photon produced from a transition n=2 to n=1.

A. $\lambda_{31} < \lambda_{21}$
B. $\lambda_{31} = \lambda_{21}$
C. $\lambda_{31} > \lambda_{21}$

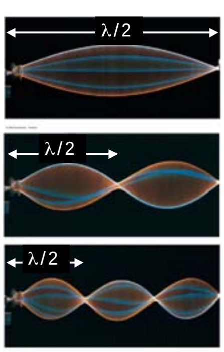
$E_{31} > E_{21}$ so $\lambda_{31} < \lambda_{21}$



But why?

- Why should only certain orbits be stable?
- Bohr had a complicated argument based on “correspondence principle”
  - That quantum mechanics must agree with classical results when appropriate (high energies, large sizes)
- But incorporating wave nature of electron gives a natural understanding of these ‘quantized orbits’

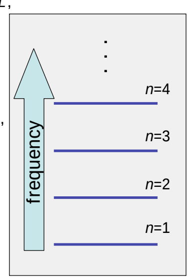
Most physical objects will vibrate at some set of natural frequencies



Fundamental, wavelength $2L/1=2L$, frequency f

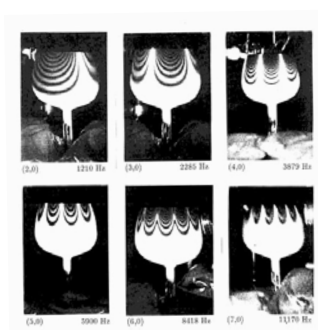
1st harmonic, wavelength $2L/2=L$, frequency $2f$

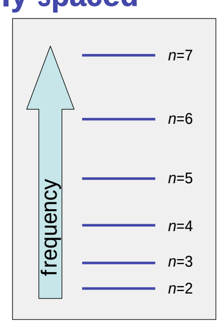
2nd harmonic, wavelength $2L/3$, frequency $3f$



Vibrational modes equally spaced in frequency

Not always equally spaced





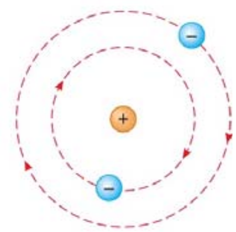
Vibrational modes unequally spaced

Why not other wavelengths?

- Waves not in the harmonic series are quickly destroyed by interference
  - In effect, the object “selects” the resonant wavelengths by its physical properties.
- Reflection from ‘end’ interferes destructively and ‘cancels out’ wave.
- Same happens in a wind instrument…
  … and in an atom!

Electron waves in an atom

- Electron is a wave.
- In the orbital picture, its propagation direction is around the circumference of the orbit.
- Wavelength = h / p (p=momentum, and energy determined by momentum)
- How can we think about waves on a circle?



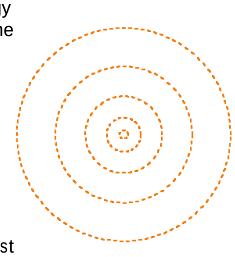
Waves on a ring

Wavelength

- Condition on a ring slightly different.
- Integer number of wavelengths required around circumference.
- Otherwise destructive interference occurs when wave travels around ring and interferes with itself.

Hydrogen atom waves

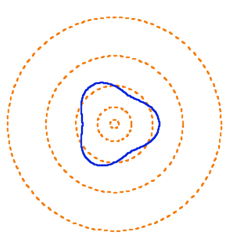
- These are the five lowest energy orbits for the one electron in the hydrogen atom.
- Each orbit is labeled by the quantum number n.
- The radius of each is na_0.
- Hydrogen has one electron: the electron must be in one of these orbits.
- The smallest orbit has the lowest energy. The energy is larger for larger orbits.



Hydrogen atom music

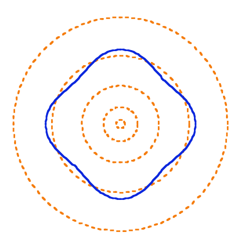
- Here the electron is in the n=3 orbit.
- Three wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its third highest note.
- Highest note (shortest wavelength) is n=1.

$$\lambda \propto r \propto na_0$$



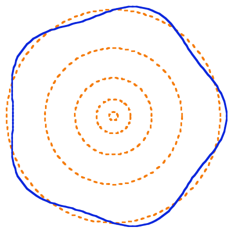
Hydrogen atom music

- Here the electron is in the n=4 orbit.
- Four wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its fourth highest note (lower pitch than n=3 note).



Hydrogen atom music

- Here the electron is in the n=5 orbit.
- Five wavelengths fit along the circumference of the orbit.
- The hydrogen atom is playing its next lowest note.
- The sequence goes on and on, with longer and longer wavelengths, lower and lower notes.



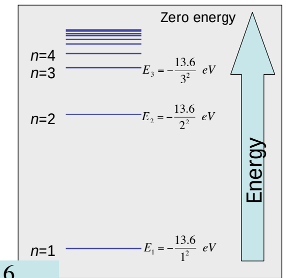
Hydrogen atom energies

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2\ m_0\ E_{kinetic}}}$$

$$E_{kinetic} = \frac{(hc)^2}{2m_0\lambda^2} \qquad \lambda \propto n$$

- Wavelength gets longer in higher n states and the kinetic energy goes down (electron moving slower)
- Potential energy goes up more quickly, also:

$$E_{pot} \propto \frac{1}{r^2} \propto \frac{1}{n^2} \qquad E_n = -\frac{13.6}{n^2}\ eV$$



Hydrogen atom question

Here is Peter Flanary's sculpture 'Wave' outside Chamberlin Hall. What quantum state of the hydrogen atom could this represent?

A. n=2
B. n=3
C. n=4

Another question

Here is Donald Lipski's sculpture 'Nail's Tail' outside Camp Randall Stadium.
What could it represent?

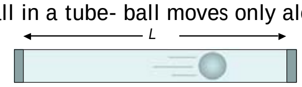
A. A pile of footballs
B. "I hear its made of plastic. For 200 grand, I'd think we'd get granite"
- Tim Stapleton (Stadium Barbers)
C. "I'm just glad it's not my money"
- Ken Kopp (New Orlean's Take-Out)
D. Amazingly physicists make better sculptures!

General aspects of Quantum Systems

- System has set of quantum states, labeled by an integer (n=1, n=2, n=3, etc)
- Each quantum state has a particular frequency and energy associated with it.
- These are the only energies that the system can have: the energy is quantized
- Analogy with classical system:
  - System has set of vibrational modes, labeled by integer fundamental (n=1), 1st harmonic (n=2), 2nd harmonic (n=3), *etc*
  - Each vibrational mode has a particular frequency and energy.
  - These are the only frequencies at which the system resonates.

Example: 'Particle in a box'

Particle confined to a fixed region of space
e.g. ball in a tube- ball moves only along length L

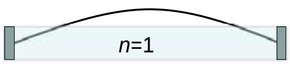


- **Classically**, ball bounces back and forth in tube.
  - No friction, so ball continues to bounce back and forth, retaining its initial speed.
  - This is a 'classical state' of the ball. A different classical state would be ball bouncing back and forth with different speed.
  - Could label each state with a speed, **momentum**=(mass)x(speed), or kinetic energy.
  - Any momentum, energy is possible. Can increase momentum in arbitrarily small increments.

Quantum Particle in a Box

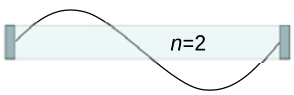
- In Quantum Mechanics, ball represented by wave
  - Wave reflects back and forth from the walls.
  - Reflections cancel unless wavelength meets the standing wave condition:
    integer number of half-wavelengths fit in the tube.

$\lambda = 2L$
One half-wavelength



momentum
$p = \frac{h}{\lambda} = \frac{h}{2L} = p_o$

$\lambda = L$
Two half-wavelengths



momentum
$p = \frac{h}{\lambda} = \frac{h}{L} = 2p_o$

Particle in box question

A particle in a box has a mass m.
It's energy is all energy of motion = $p^2/2m$.
We just saw that it's momentum in state n is np_o.
It's energy levels

A. are equally spaced everywhere
B. get farther apart at higher energy
C. get closer together at higher energy.

Quantized energy levels

- Quantized momentum

$$p = \frac{h}{\lambda} = n\frac{h}{2L} = np_o$$

- Energy = kinetic

$$E = \frac{p^2}{2m} = \frac{(np_o)^2}{2m} = n^2 E_o$$

- Or Quantized Energy

$$E_n = n^2 E_o$$