From Last Time…

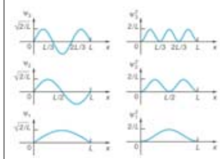
- **Hydrogen atom in 3D**
  - Electron has a particle and wave nature and is spread out over space
  - Wave nature must interfere constructively to exist
  - Satisfies 3 conditions for constructive interference

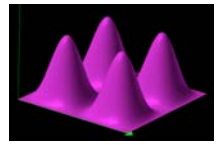
Today

- Meaning of the hydrogen atom quantum numbers
- Quantum jumps and tunneling

HW #8: Chapter 14: Conceptual: # 10, 24, 29, 33
Problems: # 2, 5
Due: Nov 15th

Particle in a box or a sphere





- Simple in 1D(or 2,3D) box
  - Fit n half wavelengths in the box
- More complex in the hydrogen atom
  - Box, the force that keeps the electron near the nucleus, is the coulomb force
  - Coulomb force is spherically symmetric - the same in any direction
  - Still 3 quantum numbers

Hydrogen Quantum Numbers

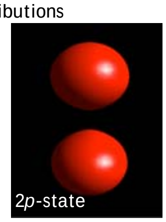
- Quantum numbers, n, l, m_l
- What do they mean?
- n: how charge is distributed radially around the nucleus. Average radial distance.
  - This determines the energy since it's dependent on the potential energy of the coulomb force and the wavelength (how many fit around)

n = 1

n = 1

Hydrogen Quantum Numbers

- Quantum numbers, n, l, m_l
- l: how spherical the charge is the distribution
  - l = 0, spherical, l = 1 less spherical…
  - n must be bigger than 1, need more room for non spherical distributions



$n = 2,\ \ell = 1,\ m_\ell = 0$

$n = 2,\ \ell = 1,\ m_\ell = \pm 1$

Hydrogen Quantum Numbers

- Quantum numbers, n, l, m_l
- n: rotation of the charge
  - If the charge is distributed such that it can rotate around the nucleus does it rotate clockwise, counterclockwise and how fast?
  - n > 1 and n > 0
  - Need a non spherical distribution
  - Need a clear axis to spin around

$n = 2,\ \ell = 1,\ m_\ell = 0$

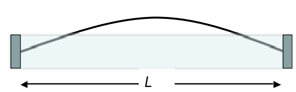
$n = 2,\ \ell = 1,\ m_\ell = \pm 1$

Uncertainty in Quantum Mechanics

Position uncertainty = L

Momentum ranges from $-\frac{h}{\lambda}$ to $+\frac{h}{\lambda}$:range $= 2\frac{h}{\lambda} = \frac{h}{L}$ (*Since λ=2L*)

$\lambda = 2L$
One half-wavelength



Reducing the box size reduces position uncertainty, but the momentum uncertainty goes up!

The product is constant:
(position uncertainty)x(momentum uncertainty) ~ *h*

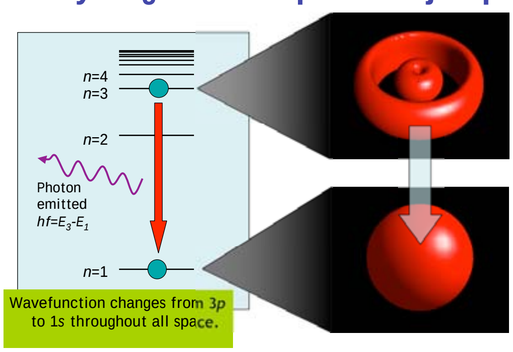
More unusual aspects of quantum mechanics

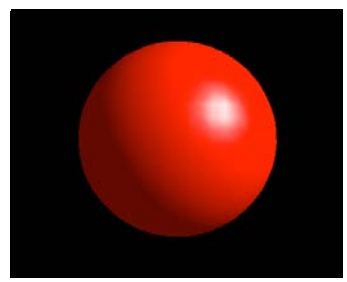
- **Quantum jumps**: wavefunction of particle changes throughout all space when it changes quantum state.
- **Superposition**: quantum mechanics says wavefunction can be in two very different configurations, both at the same time.
- **Measurements:** The act of measuring a quantum system can change its quantum state
- **Quantum Tunneling:** particles can sometimes escape the quantum boxes they are in
- **Entanglement**: two quantum-mechanical objects can be intertwined so that their behaviors are instantly correlated over enormous distances.

The wavefunction and quantum 'jumps'

- A quantum system has only certain discrete quantum states in which it can exist.
- Each quantum state has distinct wavefunction, which extends throughout all space
- It's square gives probability of finding electron at a particular spatial location.
- When particle changes it's quantum state, wavefunction throughout all space changes.

Hydrogen atom quantum jump



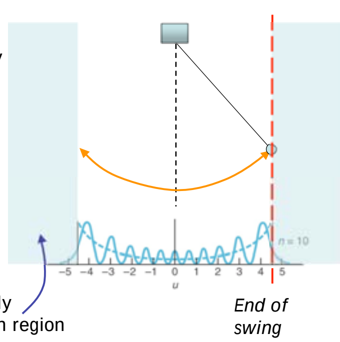


The electron jumps from one quantum state to another, changing its wavefunction everywhere.

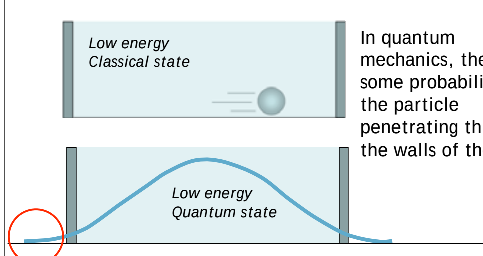
During the transition, we say that the electron is briefly in a superposition between the two states.

Unusual wave effects

- Classically, pendulum with particular energy never swings beyond maximum point.
- This region is 'classically forbidden'
- Quantum wave function extends into classically forbidden region.



Quantum mechanics says something different!

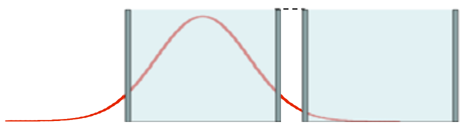


In quantum mechanics, there is some probability of the particle penetrating through the walls of the box.

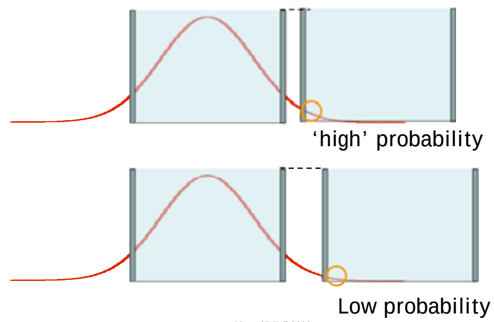
Nonzero probability of being outside the box!

Two neighboring boxes

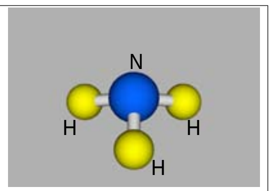
- When another box is brought nearby, the electron may disappear from one well, and appear in the other!
- The reverse then happens, and the electron oscillates back an forth, without 'traversing' the intervening distance.



The tunneling distance



Example: Ammonia molecule

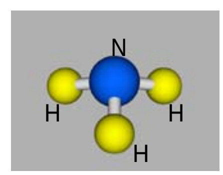


- NH_3
- Nitrogen (N) has two equivalent 'stable' positions.
- It quantum-mechanically tunnels between $2.4\text{x}10^{11}$ times per second (24 GHz)
- Was basis of first 'atomic' clock (1949)

Atomic clock question

Suppose we changed the ammonia molecule so that the distance between the two stable positions of the nitrogen atom INCREASED. The clock would

A. slow down.
B. speed up.
C. stay the same.



Classical particle in a box

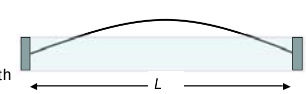
- Box is stationary, so average speed is zero.
- But remember the classical version

- Particle bounces back and forth.
  - On average, velocity is zero.
  - But not instantaneously
  - Sometimes velocity is to left, sometimes to right

Quantum version

- Quantum state is both velocities *at the same time*

$\lambda = 2L$
One half-wavelength



momentum
$|p| = \frac{h}{\lambda} = \frac{h}{2L}$

- Ground state is a standing wave, made equally of
  - Wave traveling right (positive momentum $+h/\lambda$)
  - Wave traveling left (negative momentum $-h/\lambda$)

Quantum ground state is equal superposition of two very different motions.

Making a measurement

Suppose you measure the speed (hence, momentum) of the quantum particle in a tube. How likely are you to measure the particle moving to the left?

A. 0% (never)

B. 33% (1/3 of the time)

C. 50% (1/2 of the time)

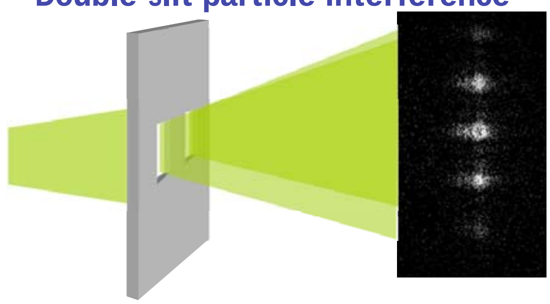
The wavefunction

- Wavefunction = Ψ
  = |moving to right> + |moving to left>
- The wavefunction for the particle is an equal 'superposition' of the two states of precise momentum.
- When we measure the momentum (speed), we find one of these two possibilities.
- Because they are equally weighted, we measure them with equal probability.

A Measurement

- We interpret this as saying that before the measurement, particle exists equally in states
  - momentum to right
  - momentum to left
- When we measure the momentum, we get a particular value (right or left).
- The probability is determined by the weighting of the quantum state in the wavefunction.
- The measurement has altered the wavefunction. The wavefunction has 'collapsed' into a definite momentum state.

Double-slit particle interference



- With single photons at a time
- Which slit does the photon go through?

Which slit?

In the two-slit experiment with one photon, which slit does the photon go through?

A. Left slit

B. Right slit

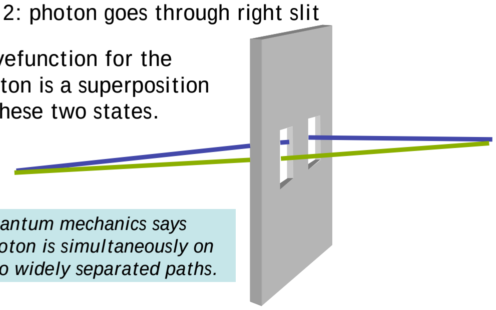
C. Both slits

Photon on both paths

Path 1: photon goes through left slit

Path 2: photon goes through right slit

Wavefunction for the photon is a superposition of these two states.

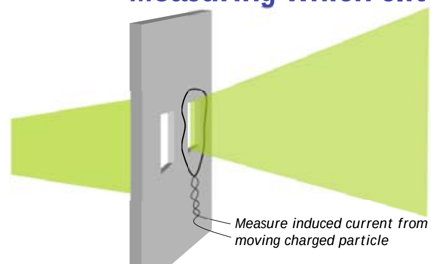


Quantum mechanics says photon is simultaneously on two widely separated paths.

Superposition of quantum states

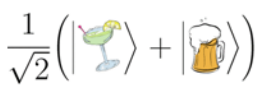
- We made a localized state made by superimposing ('adding together') states of different wavelength (momenta).
- Quantum mechanics says this wavefunction physically represents the particle.
- The amplitude squared of each contribution is the probability that a measurement will determine a particular momentum.
- Copenhagen interpretation says that before a measurement, all momenta exist. Measurement 'collapses' the wavefunction into a particular momentum state (this is the measured momentum).

Measuring which slit



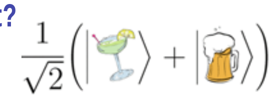
- Suppose we **measure** which slit the particle goes through?
- Interference pattern is destroyed!
- Wavefunction changes instantaneously over entire screen when measurement is made.

A superposition state



- Margarita or Beer?
- This QM state has equal superposition of two.
- Each outcome (drinking margarita, drinking beer) is equally likely.
- Actual outcome not determined until measurement is made (drink is tasted).

What is object before the measurement?



- What is this new drink?
- Is it really a physical object?
- Exactly how does the transformation from this object to a beer or a margarita take place?
- This is the collapse of the wavefunction.

Not universally accepted

- Historically, not everyone agreed with this interpretation.
- Einstein was a notable opponent
  - 'God does not play dice'
- These ideas hotly debated in the early part of the 20th century.
- However, led us to the last piece necessary to understand the hydrogen atom