From Last Time...

- Hydrogen atom quantum numbers
- Quantum jumps, tunneling and measurements

Today

- Superposition of wave functions
- Indistinguishability
- Electron spin: a new quantum effect
- The Hydrogen atom and the periodic table

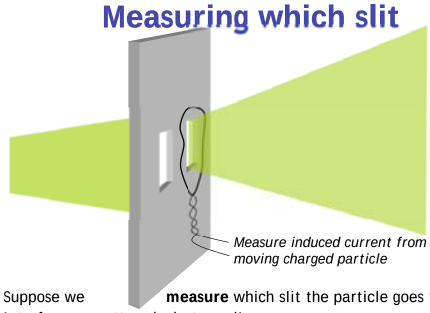
Hydrogen Quantum Numbers

- Quantum numbers, n, l, m_l
- n: how charge is distributed radially around the nucleus. Average radial distance.
  - This determines the energy
- l: how spherical the charge distribution
  - l = 0, spherical, l = 1 less spherical…
- m_l: rotation of the charge around the z axis
  - Rotation clockwise or counterclockwise and how fast
- Small energy differences for l and m_l states

$n=1,\ \ell=0,\ m_l=0$

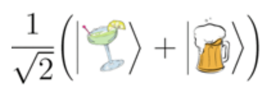
$n=2,\ \ell=1,\ m_l=\pm 1$

Measuring which slit



- Suppose we **measure** which slit the particle goes through?
- Interference pattern is destroyed!
- Wavefunction changes instantaneously over entire screen when measurement is made.
- Before superposition of wavefunctions through both slits. After only through one slit.

A superposition state



- Margarita or Beer?
- This QM state has equal superposition of two.
- Each outcome (drinking margarita, drinking beer) is equally likely.
- Actual outcome not determined until measurement is made (drink is tasted).

What is object before the measurement?

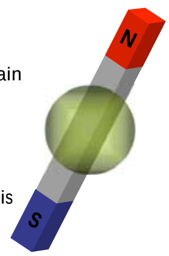
- What is this new drink?
- Is it really a physical object?
- Exactly how does the transformation from this object to a beer or a margarita take place?
- This is the collapse of the wavefunction.
- Details, probabilities in the collapse, depend on the wavefunction, and sometimes the measurement

Not universally accepted

- Historically, not everyone agreed with this interpretation.
- Einstein was a notable opponent
  - 'God does not play dice'
- These ideas hotly debated in the early part of the 20th century.
- However, one more set of crazy ideas needed to understand the hydrogen atom and the periodic table.

Spin: An intrinsic property

- Free electron, by itself in space, not only has a charge, but also acts like a bar magnet with a N and S pole.
- Since electron has charge, could explain this if the electron is spinning.
- Then resulting current loops would produce magnetic field just like a bar magnet.
- But as far as we can tell the electron is not spinning



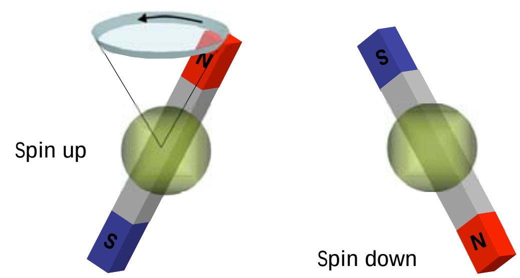
Electron magnetic moment

- Why does it have a magnetic moment?
- It is a property of the electron in the same way that charge is a property.
- But there are some differences.
  - Magnetic moment is a vector: has a size ***and*** a direction
  - It's size is **intrinsic to the electron**
  - but the direction is variable.
  - The 'bar magnet' can point in different directions.

Quantization of the direction

- But like everything in quantum mechanics, this magnitude and direction are quantized.
- And also like other things in quantum mechanics, if magnetic moment is very large, the quantization is not noticeable.
- But for an electron, the moment is very small.
  - The quantization effect is very large.
  - In fact, there is only one magnitude and two possible directions that the bar magnet can point.
  - We call these spin up and spin down.
  - Another quantum number: spin up: +1/2, down -1/2

Electron spin orientations



These are two different quantum states in which an electron can exist.

Other particles

- Other particles also have spin
- The proton is also a spin 1/2 particle.
- The neutron is a spin 1/2 particle.
- The photon is a spin 1 particle.
- The graviton is a spin 2 particle.

Particle in a box

$\lambda = 2L$
One half-wavelength

momentum $|p| = \frac{h}{\lambda} = \frac{h}{2L}$

- We labeled the quantum states with an integer
- The lowest energy state was labeled n=1
- This labeled the spatial properties of the wavefunction (wavelength, etc)
- Now we have an additional quantum property, spin.
  - Spin quantum number could be +1/2 or -1/2

There are two quantum states with n=1
Can write them as $|n=1,\ spin=+1/2\rangle$ $|n=1,\ spin=-1/2\rangle$

Spin 1/2 particle in a box

We talked about two quantum states

$|n=1,\ spin=+1/2\rangle$ $\quad$ $|n=1,\ spin=-1/2\rangle$

In isolated space, which has lower energy?

A. $|n=1,\ spin=+1/2\rangle$

B. $|n=1,\ spin=-1/2\rangle$

C. Both same

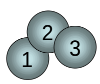
An example of degeneracy: two quantum states that have exactly the same energy.

Indistinguishability

- Another property of quantum particles
  - All electrons are ABSOLUTELY identical.
- Never true at the macroscopic scale.
- On the macroscopic scale, there is always *some* aspect that distinguishes two objects.
- Perhaps color, or rough or smooth surface
- Maybe a small scratch somewhere.
- Experimentally, no one has ever found **any** differences between electrons.

Indinstinguishability and QM

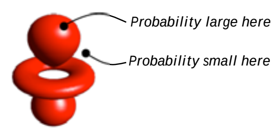
- Quantum Mechanics says that electrons are absolutely indistinguishable.
  Treats this as an experimental fact.
  - For instance, it is impossible to follow an electron throughout its orbit in order to identify it later.
- We can still label the particles, for instance
  - Electron #1, electron #2, electron #3



- But the results will be meaningful only if we preserve indistinguishability.
- Find that this leads to some unusual consiquenses

Example: 2 electrons on an atom

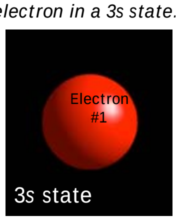
- Probability of finding an electron at a location is given by the square of the wavefunction.

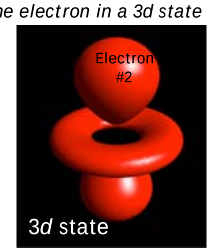


- We have two electrons, so the question we would is ask is
  - How likely is it to find one electron at location r_1 and the other electron at r_2?

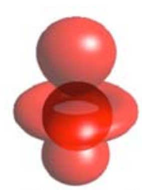
- Suppose we want to describe the state with

one electron in a 3s state… *and one electron in a 3d state*





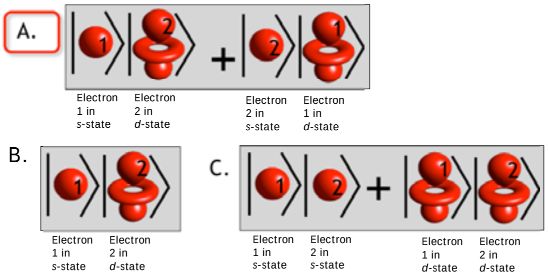
On the atom, they look like this. (Both on the same atom).



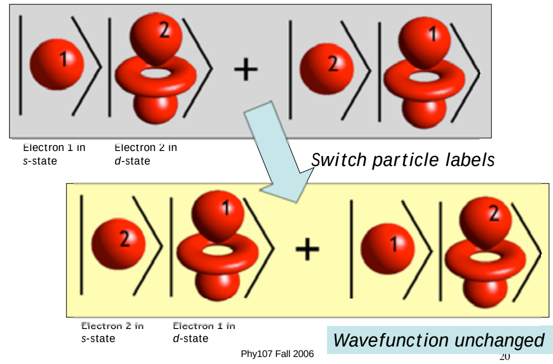
- Must describe this with a wavefunction that says
  - We have two electrons
  - One of the electrons is in *s*-state, one in *d*-state
- Also must preserve indistinguishability

Question

Which one of these states doesn't 'change' when we switch particle labels.



Preserves indistinguishability



Physically measurable quantities

- How can we label particles, but still not distinguish them?
- What is really meant is that no physically measurable results can depend on how we label the particles.
- One physically measurable result is the probability of finding an electron in a particular spatial location.

Probabilities

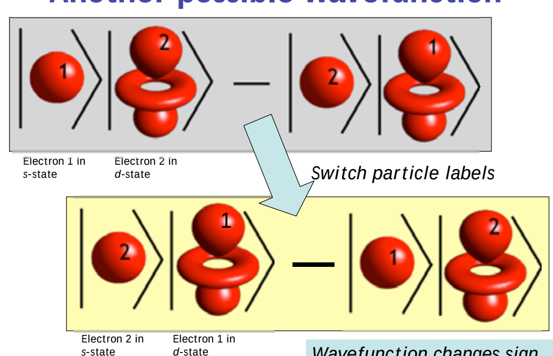
- The probability of finding the particles at particular locations is the square of the wavefunction.
- Indistinguishability says that these probabilities cannot change if we switch the labels on the particles.
- However the wavefunction *could* change, since it is not directly measurable. (*Probability is the square of the wavefunction)*

Two possible wavefunctions

- Two possible **symmetries** of the wavefunction, that keep the probability unchanged when we exchange particle labels:
  - The wavefunction does not change **Symmetric**
  - The wavefunction changes sign only **Antisymmetric**

In both cases the square is unchanged

Another possible wavefunction

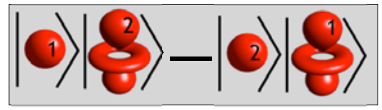


Spin-statistics theorem

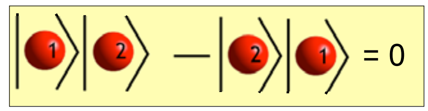
- In both cases the probability is preserved, since it is the square of the wavefunction.
- Can be shown that
  - Integer spin particles (e.g. photons) have wavefunctions with '+' sign (symmetric) These types of particles are called **Bosons**
  - Half-integer spin particles (e.g. electrons) have wavefunctions with '-' sign (antisymmetric) These types of particles are called **Fermions**

So what?

- Fermions - antisymmetric wavefunction:



Try to put two Fermions in the same quantum state (for instance both in the *s*-state)



Pauli exclusion principle

- Only wave function permitted by indistinghishability is exactly zero. This means that this never happens.
- Cannot put two Fermions in same quantum state
- This came entirely from indisinguishability, that electrons are identical.
- Without this,
  - there elements would not have diff. chem. props.,
  - properties of metals would be different,
  - neutron stars would collapse.

Include spin

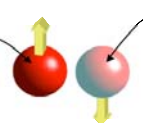
- We labeled the states by their quantum numbers. One quantum number for each spatial dimension.
- Now there is an extra quantum number: spin.
- A quantum state is specified by it's space part and also it's spin part.
- An atom with several electrons filling quantum states starting with the lowest energy, filling quantum states until electrons are used.

Putting electrons on atom

- Electrons are Fermions
- Only one electron per quantum state

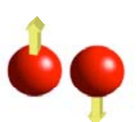
occupied *unoccupied*

Hydrogen: 1 electron
one quantum states occupied



n=1 states

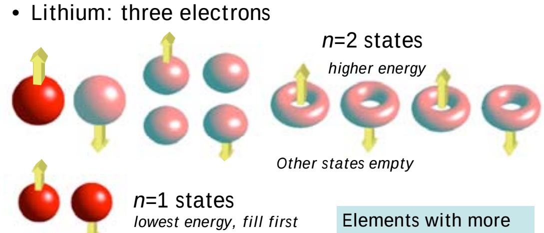
Helium: 2 electrons
two quantum states occupied



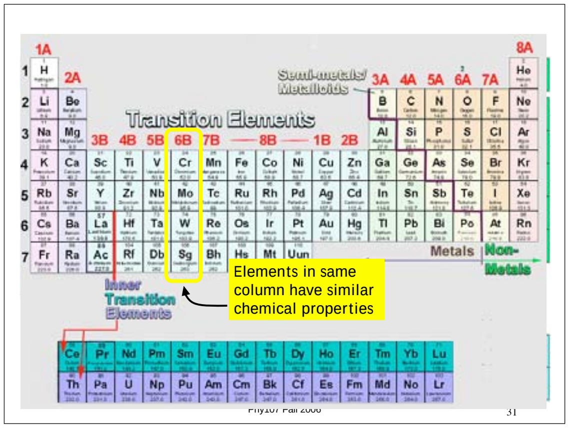
n=1 states

Other elements

- More electrons requires next higher energy states
- Lithium: three electrons



Elements with more electrons have more complex states occupied


1A
2A
8A
3A 4A 5A 6A 7A
3B 4B 5B 6B 7B 8B 1B 2B
Transition Elements
Semi-metals/ Metalloids
Metals
Non-Metals
Inner Transition Elements
Elements in same column have similar chemical properties
Phy107 Fall 2006
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