EXPERIMENT 6: TRANSIENT RESPONSE OF A SERIES LCR CIRCUIT 10/14/03

In this experiment we will study the transient response of a series LCR circuit in the critically damped, overdamped, and underdamped cases. For the overdamped case we will determine the decay constant, while for the underdamped case we will measure the natural frequency, the damping constant, and the Q factor.

Before coming to the laboratory make sure you understand the following material. For the circuit shown below, suppose the switch S has been closed for a long time so that v_C , v_L , and v_R have all reached their asymptotic values. What values will v_C , v_L and v_R take on at this point? (Write the answer and a brief explanation in your notebook.) Now suppose that at time t = 0 the switch is opened. Write down the initial conditions for the current i(t) and its time derivative di/dt just after the svitch opens.

When the switch is open the current flowing in the circuit must satisfy the differential equation:

$$\frac{\mathrm{d}^2 \mathrm{i}}{\mathrm{d}t^2} + \frac{\mathrm{R}}{\mathrm{L}}\frac{\mathrm{d}\mathrm{i}}{\mathrm{d}t} + \frac{1}{\mathrm{LC}}\mathrm{i} = 0.$$

The solutions to this equation will be discussed in class, and can also be found in some text books. In your notebook you should write down the solutions to the differential equation for i(t) as a function of the time for the underdamped, critically damped, and overdamped cases. In addition, write down expressions for v_R , v_L and v_C for the underdamped case.



All measurements are to be made with an oscilloscope using X10 attenuator probes. Note that by using values of R that are too small you can obtain circuits with very high Q's and high voltages (that blow out scopes and cause shocks). In particular be careful that you do not inadvertently set R to zero.

Construct the circuit as shown in the diagram above. Use a DC power supply set to about 3 V for V₀. Do not ground either terminal of the power supply (the circuit will be grounded at various points depending on how you attach the scope). Adjust the <u>current</u> control on the power supply to limit the current to about 0.5 A (this can be done by simply shorting the + and – terminals together and reading the current meter on the supply). The switch is a mercury wetted relay which is driven by the AC power lines (use terminals 3 and 4). The switch opens and closes 60 times a second. Use decade boxes for R, L, and C. Note that on the capacitance decade boxes, MF means 10^{-6} F.

- 1. Set L = 10 mH, C = 100 nF and $R = 50 \Omega$. Make rough sketches of $v_R(t)$, $v_L(t)$ and $v_C(t)$ for one complete cycle of the switch. (NOTE: We adopt the convention that v_R , v_L and v_C are all to be measured in the counter-clockwise sense around the LCR loop, as shown in the circuit diagram.) Indicate on your sketches the points at which the switch opens and closes. What are the observed asymptotic values of v_R , v_L and v_C for the switch-closed portion of the cycle?
- 2. When the switch is closed, the capacitor charges up quickly and thus v_c reaches its equilibrium value in a very short time. However, v_R rises to its asymptotic value much more slowly. Make a rough measurement of the time it takes for v_R to come to $(1 e^{-1})V_0$, where V_0 is its asymptotic value, and compare your measurement with the expected time constant.

In the remaining sections we will focus on what happens during the switch-open part of the cycle.

3. (a) For a fixed value of L and C, experimentally determine the critical damping resistance by observing v_R vs time on the scope. Compare the measured and calculated values of the critical damping resistance.

(b) Determine v_R as a function of t (out to at least $V_0/20$) (with R at the critical damping value) as accurately as you can from the scope. (This is a bit tricky to do. The main problem is that di/dt is zero at t = 0, which makes it difficult to get the scope to trigger right at t = 0. Try using v_L or v_C to trigger to scope.) Record the results in a table and make a graph of v_R vs t on semilog graph paper with v_R plotted on the logarithmic axis. Compare your measurements of v_R with the theoretical prediction, $v_R(t) = V_0 (1 + \gamma t) \exp(-\gamma t)$, where $\gamma = R/2L$, and R is the value used in your circuit.

4. (a) Next increase R by a factor of 5 or 10 to produce overdamped behavior. Measure and tabulate v_R as a function of t and graph the results (on semilog paper).

(b) Use a ruler to draw a straight line through the measured points and determine the decay constant, γ . Compare your measured decay constant with the expected value,

$$\gamma = (R/2L) [1 - (1 - (4L/R^2C))^{1/2})].$$

5. (a) Choose values for R, L, and C to produce underdamped behavior with a period T $\approx 60 \,\mu s$ and with Q ≈ 6 .

Make <u>careful</u> sketches or photographs of v_R , v_L and v_C as a function of time from t = 0 out to $t = 250 \ \mu$ s. Make sure that you get the relative phase of the signals right and that the signs are correct $(v_L + v_R + v_C \text{ should be zero for all t})$.

(b) Make a list of the zero crossing times of $v_R(t)$ and determine the period and angular frequency (ω) of the oscillation. Compare your result with the expected value,

$$\omega = [(1/LC) - (R^2/4L^2)]^{1/2}.$$

(c) Make a list of the extrema (both positive and negative) of v_c . Plot the resulting values of $|v_c|$ as a function of t (on semilog paper) and determine the decay constant γ . Compare your result with the expected value, $\gamma = R/2L$.

(d) The Q of the circuit can be determined from the magnitude of the first extremum in v_L . Note that the formula for v_L can be written in the form:

$$v_{L}(t) = -V_{0}Q \left[1 + \left(\frac{\gamma}{\omega}\right)^{2}\right] \exp(-\gamma t)\sin\omega t$$
.

At the first extremum ($\omega t = \pi/2$) we have:

$$v_{\rm L} = -V_0 Q \left[1 + \left(\frac{\gamma}{\omega}\right)^2 \right] \exp(-\frac{\pi\gamma}{2\omega}).$$

Use this formula and your measured values of γ and ω to determine Q. Compare your result with the expected value, Q = $\omega L/R$ where the formula for ω is given in (b) above.