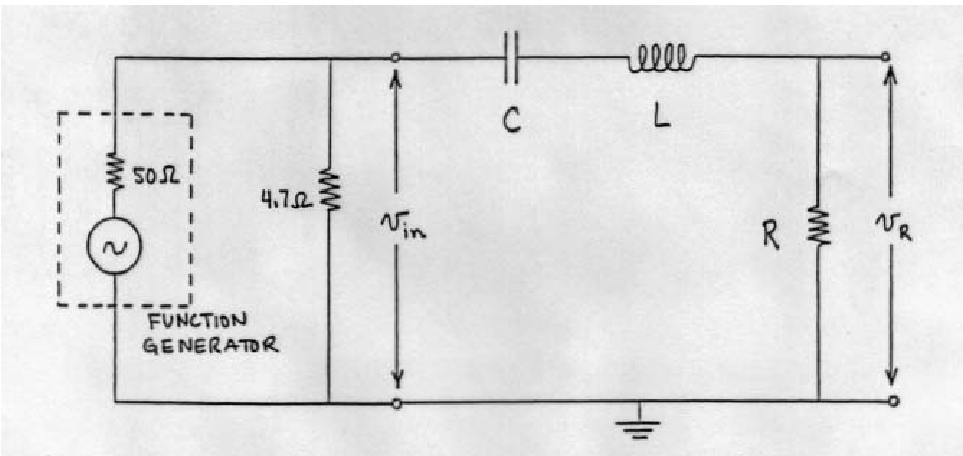
EXPERIMENT 7: FOURIER ANALYSIS OF A SQUARE WAVE 10/13/03

In this lab we will experimentally determine the Fourier components of a square wave. The basic idea of the experiment is that one can pick out an individual Fourier component of the square wave by using a resonant RLC filter whose gain is close to 1 at the frequency of the Fourier component we want to observe, and close to zero at other frequencies. This means that we need to use a filter circuit with a high Q. In addition we will need to carefully adjust the circuit elements to make sure that the resonant frequency matches the frequency of the Fourier component we are looking at.

The circuit that will be used is shown below. The purpose of the 4.7 Ω resistor is to lower the output impedance of the voltage source (the output impedance of the function generator itself is 50 Ω).



Depending on the apparatus available, use one of the two configurations given in the table below
The circuits have Q $\simeq$ 10 at the fundamental frequency, and a resonance width of a few hundred Hz. Note that on the capacitance decade boxes, MF means 10^{-6} F. Set the function generator for about 75% of maximum output, and trigger the scope on the output of the function generator. All measurements are to be made with the scope.

L	R	f_0	Waveform
10 mH	33 Ω	5.1 kHz	Square Wave
70 mH	82 Ω	1.9 kHz	Square Wave

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The data for this experiment are to be collected in a table with the following entries: the Fourier component (i.e. the harmonic number), the frequency f_n of the harmonic, the value of C that gives resonance, v_R for each harmonic (parts 3,4,5), the values of v_{in}, v_R', and v_{in}'' for each harmonic (part 6), the gain and attenuation factors (G and A from part 6), the corrected Fourier amplitudes (part 7), the theoretical Fourier amplitudes (part 8) and the percentage errors. The first nine Fourier components will be measured.

1. Start by setting the function generator to produce sine waves of the frequency indicated in the table above. Adjust C to get the resonant frequency of the filter as close as possible to the generator frequency, and then if necessary, make slight adjustments to the generator until the frequency is right on resonance (there should be no phase shift between v_R and v_{in}). Now switch the function generator to the desired waveform (square waves) and check that v_R is still accurately in phase with v_{in}.

2. Measure the amplitude V_0 of the input square wave. For this measurement, the filter circuit should be disconnected from the voltage source. (The reason is that the filter circuit draws current and therefore loads down the voltage source, which attenuates the input signal and distorts the waveform. We will measure the attenuation in step 6 and apply appropriate corrections in the end.) Make a sketch of the input waveform and record the value of V_0. As you go through the remaining steps, be careful not to change the amplitude of the function generator.

3. Reconnect the circuit and measure the amplitude v_R of the n = 1 (fundamental) Fourier component.

4. Next measure the amplitude of the odd Fourier components (n = 3, 5, 7, 9). To do this, set the capacitance C so that the circuit resonates at frequency $f_n = nf_0$, and if necessary make small adjustments (not more than a few %) in the frequency of the function generator to obtain the proper phase between the input and output waves. (The phase is the most sensitive indicator of whether you are on resonance).

   As you make the measurements you will notice that the individual peaks that make up the output sine wave do not all have the same height (the output voltage gets a “kick” each time the input voltage changes, and then decays exponentially until the next “kick”). You should take v_R to be the average amplitude of the output wave.

   For each value of n record the amplitude v_R, and the capacitor setting. Also, make a sketch of the output waveform for n = 3 and n = 5.

5. Now measure the amplitude of the even Fourier components ($n = 2, 4, 6, 8$). The first thing to do is readjust the frequency of the function generator (as in step 1) with the capacitor set to its original value. To measure the Fourier amplitudes, you should follow the same procedure as in step 4, except that the frequency of the oscillator should not be adjusted. (The reason is that the output signals will look funny, and it will be hard to tell when you have the correct phase.) Record the values of v_R (the average amplitude) and of C for each n, and make sketches of the output waveform for $n = 2$ and $n = 4$.

6. The measured Fourier amplitudes need to be corrected for two effects. The first is the attenuation of the input wave, which is brought on by the non-zero output impedance of the voltage source. The second effect is that the gain of the filter circuit is not exactly 1 on resonance, due to the small but non-zero resistance of the inductor and capacitor boxes. To measure these correction factors, we will use the sine wave output from the function generator, and will adjust the frequency of the generator to match the frequency of the Fourier component. In other words, we will now be setting the generator frequency to f_n rather than f_0.

   For each value of n ($n = 1$-9) set the capacitor to the value you used previously, set the generator to $f_n = nf_0$, and then make small adjustments in the generator frequency until the input and output waves are in phase. Record the amplitude of the input sine wave (v_{in}') and the output wave (v_R'). Then disconnect the filter and measure the unattenuated input amplitude (v_{in}''). Record the measurements in your table, and include in the table the resulting values of the attenuation factor:

$$A = v_{in}' / v_{in}''$$

   and the gain

$$G = v_R' / v_{in}' .$$

7. Calculate and record the corrected Fourier amplitudes, v_n ,

$$v_n = \frac{v_R}{G \times A} .$$

8. Compare your results with the theoretical values predicted from the Fourier analysis of a symmetric wave.

$$v(t) = \sum_{n=1}^{\infty} v_n \sin_n t ,$$

where

$$v_n = \frac{4V_0}{n\pi} \text{ for odd n}$$

$$v_n = 0 \text{ for even n}$$

For the odd harmonics, calculate and record the percentage error. If the measurements were done carefully the errors should be about 10% or less.

9. Can you explain why the waveforms for the even n harmonics (observed in step 5) look the way they do? What would happen if you used a filter circuit with a higher Q?