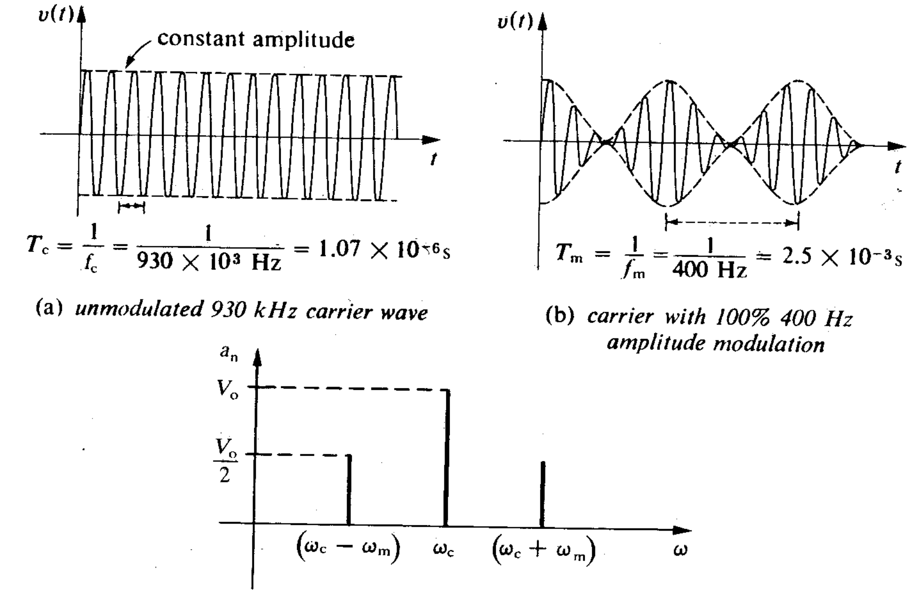


(a) *unmodulated 930 kHz carrier wave*

(b) *carrier with 100% 400 Hz amplitude modulation*

(c) *Fourier components of 100% modulated carrier of* (b)

FIGURE 3.8 Amplitude modulation.

say, from 929.9 to 930.1 kHz, a bandwidth of 0.2 kHz or 200 Hz, no 400-Hz signal would be present in the receiver.

Let us Fourier analyze the amplitude-modulated carrier.

$$v(t) = V_0(1 + M\cos\omega_m t)\cos\omega_c t \tag{3.26}$$

where $\omega_m = 2\pi f_m$, $f_m = 400$ Hz, $\omega_c = 2\pi f_c$, $f_c = 930$ kHz, and M is the degree of modulation, $0 \leqq M \leqq 1$. M is 1.0 in Fig. 3.8(b). The modulated carrier wave $v(t)$ is an even function, so only cosine terms will be present in the Fourier expansion. Also, we can see that $a_0 = 0$, because the average dc level is zero. Thus

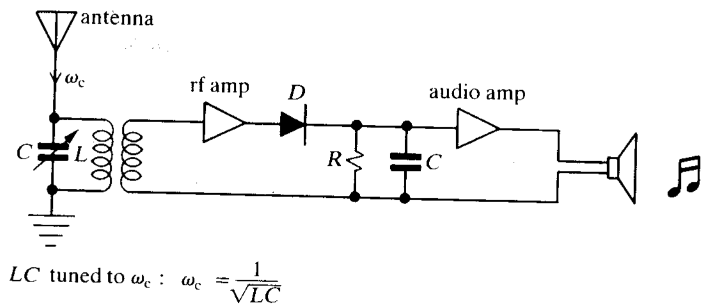
$$v(t) = \sum_{n=1}^{\infty} a_n \cos n\omega t \tag{3.27}$$

where

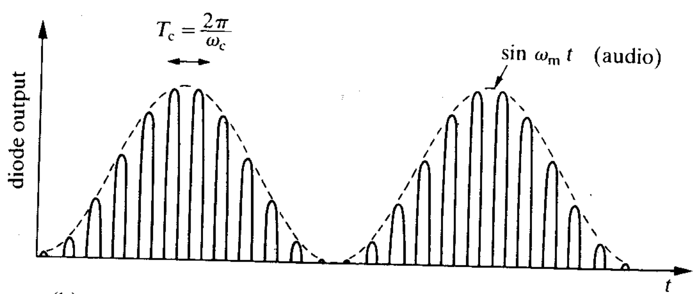
$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} v(t)\cos n\omega t\,dt \tag{3.28}$$

Choose $T = 2\pi/\omega_m \equiv T_m$; this is the shortest time interval over which $v(t)$ is periodic. Substituting (3.26) in (3.28) yields, with $\omega = \omega_m$,

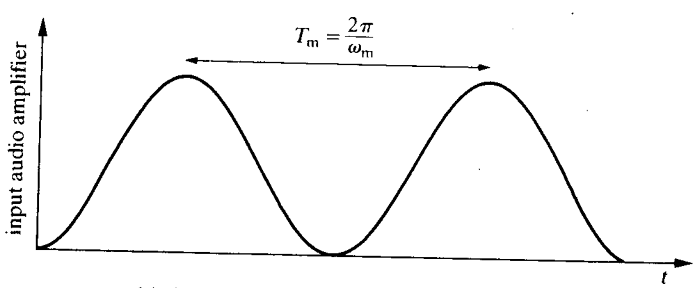
$$a_n = \frac{4}{T_m}\int_0^{T_m/2} V_0(1 + M\cos\omega_m t)\cos\omega_c t\cos n\omega_m t\,dt$$



(a) *circuit*



(b) *output of diode before passing through the RC low-pass filter*



(c) *filtered diode output (original audio signal)*

FIGURE 3.10 Simple diode AM receiver.

RC filter that passes only the audio envelope at ω_m, provided

$$T_m \gg RC \gg T_c \qquad \text{where } T_m = \frac{2\pi}{\omega_m} \text{ and } T_c = \frac{2\pi}{\omega_c}$$

Thus, the filter output is the same as the original audio modulation of the transmitted wave.

Another type of amplitude modulation widely used in long-range noncommercial radio communication is *single sideband* (SSB). In this technique only one set of sidebands is actually transmitted, not the carrier wave frequency is

$$\omega = \omega_c + \Delta\omega \cos \omega_m t \tag{3.39}$$

where $\Delta\omega$ is the maximum frequency swing (the *deviation*) of the carrier. The FM carrier wave is then

$$v_c(t) = V_0 \cos \omega t = V_0 \cos[\omega_c t + \Delta\omega \cos \omega_m t] \tag{3.40}$$

The carrier frequency swings from $\omega_c + \Delta\omega$ to $\omega_c - \Delta\omega$, as shown in Fig. 3.13. In commercial FM radio $\Delta\omega/2\pi$ is limited to 75 kHz, so the maximum carrier-frequency swing is ±75 kHz around $\omega_c/2\pi$; for example, if $\omega_c/2\pi = f_c = 100$ MHz, then the swing is from 100.075 to 99.925 MHz. Commercial FM stations are assigned frequency f_c every 200 kHz, so their signals do not overlap. Notice that the frequency swing of the carrier is determined by the *amplitude* ($\Delta\omega$) of the audio modulation, whereas the audio modulation *frequency* (ω_m) determines how often the carrier frequency swings up and

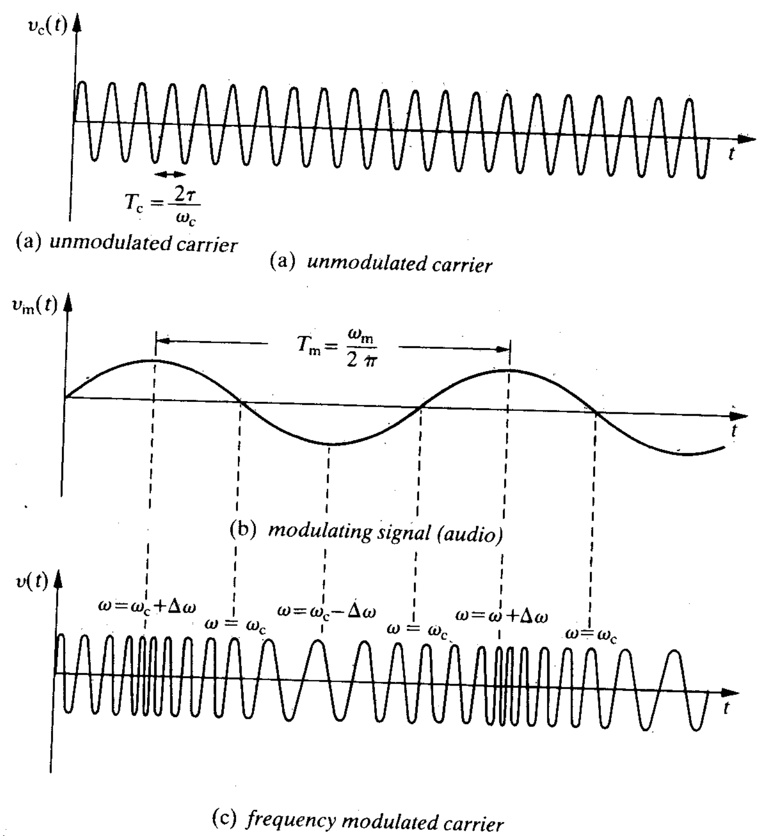


FIGURE 3.13 FM radio waveforms.