AN $SU_3$ MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)



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ABSTRACT

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number 1/3 and is fractionally charged. $SU_3$ (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. Quantitative speculations are presented concerning resonances that have not as yet been definitively classified into representations of $SU_3$. A weak interaction theory based on right and left handed aces is used to predict rates for $|\Delta S| = 1$ baryon leptonic decays. An experimental search for the aces is suggested.

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## I. INTRODUCTION

We wish to consider a higher symmetry scheme for the strongly interacting particles based on the group $SU_3$. The way in which this symmetry is broken will also concern us. Motivation, other than aesthetic, comes from an attempt to understand certain regularities, described below, in the couplings and spectra of particles and resonances.

Since we deal with the same underlying group as that of the Eightfold Way [1),2)], particle classification will be similar in the two models. However, we will find restrictions on the representations that may be used to classify particles, restrictions that are not contained in the Eightfold Way. The $(N, \Lambda, \Sigma, \Xi)$ and the pseudoscalar mesons will fall into octets; the vector mesons will be grouped into an octet and singlet, where the two representations will mix by a predictable amount when unitary symmetry is broken; while the $(\Delta_\delta(1238), \Sigma_\delta(1385), \Xi_\delta(1530), \Omega_\delta(1686))$ will form a decuplet in the usual manner. The choice of 1, 8, and 10-dimensional representations for baryons, along with 1 and 8-dimensional representations for mesons will be a natural consequence of the model.

A simple mechanism for the breaking of unitary symmetry will be presented. Mass formulae connecting members of the same representation or members of different representations will follow. The meson and baryon mass spectra will be related to each other.

The Eightfold Way does not allow a unique determination of the baryon-baryon-meson interaction. Two types of coupling, known as F and D [1)], are possible. The model we shall consider will suggest what ratio of F to D coupling is to be taken. We will also see that relations between coupling constants that govern the interactions of different representations may exist. For example, for the octet and singlet of vector mesons $(\rho, K^*, \omega_8)$ and $\omega_0$ we will find a natural connection between the amplitudes for $\omega_8 \to \rho + \pi$ and $\omega_0 \to \rho + \pi$ which will suppress the reaction $\varphi \to \rho + \pi$ by a predictable amount.

The model will allow a simple extension to include the weak interactions. The conserved vector current theory [3], and the $\Delta S/\Delta Q = +1$, $|\Delta I| = 1/2$ rules for leptonic decays will follow naturally. Rates for hyperon $\beta$- decays will be given.

We have included two sections where we quantitatively speculate about the application of our theory to resonances that have not as yet been definitively classified into representations of $SU_3$.

## II. THE MODEL

The Eightfold Way and our model differ in the way particles or resonances are constructed. In the Eightfold Way, the 8 pseudoscalar mesons may be thought of as bound states of a fundamental triplet $(p, n, \Lambda)$. For example, the $\pi^+$ would be represented by $\bar{n}p$, the $K^-$ by $\bar{p}\Lambda$, etc. In the language of group theory, the 8-dimensional representation of $SU_3$ containing the mesons is included in the 9-dimensional baryon × antibaryon cross product space, i.e., $\bar{3} \times 3 = 8 + 1$. However, if as in the Sakata model [4] we attempt to construct the baryons out of this triplet (for example : $n \sim \bar{p}pn$, $\Xi^- \sim \bar{p}\Lambda\Lambda$, etc.) we are no longer able to classify them into the familiar group of 8 particles. The difficulty stems from the fact that the eight-dimensional representation describing the baryons is not contained in the 27-dimensional antibaryon × baryon × baryon cross product space, $\bar{3} \times 3 \times 3$. In the decomposition $\bar{3} \times 3 \times 3 = 3 + 3 + \bar{6} + 15$, only the 15-dimensional representation can accommodate all 8 baryons. Unfortunately, this representation contains other particles whose masses may be predicted by the Gell-Mann - Okubo mass formula

$$m = m_0 \left\{ 1 + aY + b \left[ I(I+1) - 1/4\, Y^2 \right] \right\} . \qquad (2.1)$$

Since these particles or resonances do not seem to be present in nature, we must abandon the Sakata model and work with the 8 baryons themselves as "fundamental" units.

There is, however, another possibility based on a genuine desire to keep certain elements of the Sakata model. If we build the baryons from a triplet of particles $(p_0, n_0, \Lambda_0)$, $(p_0, n_0)$ being a strangeness zero isospin doublet and $\Lambda_0$ a strangeness $-1$ singlet, using $3 \times 3 \times 3$ instead of $\bar{3} \times 3 \times 3$ we find that classification of baryons into a set of 8 is possible since $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. We note that the 10-dimensional representation is present so that the $\Delta_\delta$ decuplet may also be constructed from our three fundamental units. The 27-dimensional representation and the $\overline{10}$-representation which occur naturally in the Eightfold Way and which do not seem to be used by nature for the baryons are suggestively absent.

The only difficulty is that now the baryons seem to have baryon number 3. This we get around by assigning baryon number 1/3 to each member of the basic triplet, which leads via the Gell-Mann - Nishijima charge formula, $Q = e\left[I_z+1/2(B+S)\right]$, to non-integral charges for $(p_0, n_0, \Lambda_0)$ [5]. The isospin doublet $(p_0, n_0)$ contains charges $(2/3, -1/3)$ while the isospin singlet $\Lambda_0$ has charge $-1/3$. We shall call $p_0, n_0$, or $\Lambda_0$ an "ace". Note that the charges of the aces are just those of $(p, n, \Lambda)$, but shifted by a unit of $-1/3$. The isospin and strangeness content, along with space-time properties, remain the same. The ace properties are summarized in Table 1. We will work with these aces as fundamental units from which all mesons and baryons are to be constructed.

Perhaps it is best to state ahead of time the point of view we hold regarding this model. $SU_3$ is the group of rotations in a three dimensional vector space (over complex numbers). The Eightfold Way singles out for special consideration objects in this space that have remarkably complicated transformation properties (second, third, and fourth rank tensors corresponding to 8, 10, and 27-dimensional representations). In a manner of speaking, the Eightfold Way is a theory based on a vector space without vectors. We focus our primary attention on vectors (aces) in this space where $SU_3$ operates. It is our hope that in so doing we will better be able to express certain symmetries and asymmetries present in nature. Whether or not these vectors correspond to physical particles is of course impossible to say.

The validity of many of our results may not be taken as direct evidence for the existence of aces, at least not if we are to believe that the world is as complicated as most modern theories make it out to be. For example, baryon mass formulae will be obtained by a linear treatment of the aces; but particle physics has taught us that linearity of this type should be most unreasonable. On the other hand, saying that the vectors or aces are some kind of spurions, fictional particles that help in computing consequences of symmetry, is also not correct. Aces, unlike conventional spurions, bind and have physically observable mass differences. The model we shall consider is quite peculiar. It is too simple to be literally valid, yet too complex to be understood in conventional terms.

## III. THE BARYONS

For convenience, let us designate the aces $(p_o, n_o, \Lambda_o)$ by $(A_1, A_2, A_3)$. In order to construct the states representing the eight baryons we consider the reduction of the 27-dimensional cross product space of "treys" $A_a A_b A_c$ [6] $(a,b,c = 1,2,3)$ into irreducible representations [7].

$$A_a \times A_b \times A_c \sim T_{abc,} + T_{ab,c} + T_{ac,b} + T_{a,b,c}$$

$$3 \times 3 \times 3 \sim 10 + 8 + 8 + 1 \tag{3.1}$$

Here $T_{abc,}$ is totally symmetric in its indices and will represent members of the $\Delta_\delta$ (1238) decuplet, while $T_{ab,c}$ is symmetric in $a,b$ : being explicitly given by

$$T_{ab,c} = \frac{1}{2\sqrt{2}} (T_{abc} - T_{acb} + T_{bac} - T_{bca}) \tag{3.2}$$

and will be taken to represent the nucleon octet ($T_{ac,b}$ could of course be used just as well). $T_{a,b,c}$ is totally antisymmetric in $a,b,c$ and allows for the existence of an $I = 0$, $S = -1$ singlet to be identified with the $\Lambda_\beta$ (1405). The fact that the $\Delta_\delta$ does not seem to belong to the 27-dimensional representation [8] or the $\overline{10}$-representation of $SU_3$ may be taken as a prediction of this model.

### A. The Baryon Octet

We now list the 8 baryon states :

$$
\begin{aligned}
p &= T_{11,2} = -2T_{12,1} = 1/\sqrt{2}(T_{112}-T_{121}) \\
n &= -T_{22,1} = 2T_{12,2} = 1/\sqrt{2}(T_{212}-T_{221}) \\
\Lambda^0 &= -\sqrt{2/3}(T_{13,2}-T_{23,1}) = 1/\sqrt{12}(T_{123}-T_{213}+T_{231}-T_{132}+2T_{321}-2T_{312}) \\
\Sigma^+ &= -T_{11,3} = 2T_{13,1} = 1/\sqrt{2}(T_{131}-T_{113}) \\
\Sigma^0 &= 2\,T_{12,3} = -\sqrt{2}\,(T_{13,2}+T_{23,1}) = 1/2(T_{123}+T_{213}-T_{132}-T_{231}) \qquad (3.2) \\
\Sigma^- &= T_{22,3} = -2T_{23,2} = 1/\sqrt{2}(T_{223}-T_{232}) \\
\Xi^0 &= T_{33,1} = -2T_{13,3} = 1/\sqrt{2}(T_{331}-T_{313}) \\
\Xi^- &= -T_{33,2} = 2T_{23,3} = 1/\sqrt{2}(T_{323}-T_{332})
\end{aligned}
$$

[9]

For example, by inspection of the subscripts, $T_{22,3} = \Sigma^-$ has $I_z = (-1/2)+(-1/2)+0$ and strangeness $S = 0+0+(-1)$. Note that the non-integral ace charges $(2/3,-1/3,-1/3)$ are forced on us when we assume that the baryons are constructed from the aces as in (3.2). In the limit of unitary symmetry the 3 aces are indistinguishable and all baryon states have the same structure and mass. This is represented in Fig. 1a, where we have drawn the unitary symmetric limit of the trey $T_{abc}$ from which the octet members are constructed.

The mass $m(T_{ab,c})$ of a baryon $T_{ab,c} = (1/2\sqrt{2})(T_{abc}-T_{acb}+T_{bac}-T_{bca})$ may be thought of as the average of the masses of $T_{abc}$, $T_{acb}$, $T_{bac}$, and $T_{bca}$. We represent the mass $m(T_{abc})$ of $T_{abc}$ by

$$m(T_{abc}) = m(a)+m(b)+m(c)-E^8_{ab.}-E^8_{a.c}-E^8_{.bc}$$

where $m(a)$ is the mass of the ace $a$ and the $E^8$'s are octet binding energies[10]. The baryon masses are given explicitly in Appendix A. In the limit of unitary symmetry we have $m(a)=m(b)=m(c)$, $E^8_{ab.}=E^8_{cd.}$, $E^8_{a.b}=E^8_{c.d}$, $E^8_{.ab}=E^8_{.cd}$ so that the masses of all the baryons are identical.

We now assume that unitary symmetry is broken due to the fact that the singlet $A_3$ is heavier than the doublet $(A_1,A_2)$ [11], in analogy to the Sakata model where the $\Lambda$ was assumed heavier than the $(p,n)$. The baryons now break up into distinguishable groups, so that instead of Fig. 1a, we have Fig. 2. As a first approximation, neglecting differences in binding energies, we immediately find that the baryons increase their masses linearly with strangeness, i.e. [12],

$$\underset{(1115)}{m(\Lambda)} \approx \underset{(1193)}{m(\Sigma)} \quad , \quad \underset{(1154)}{(m(\Sigma)+m(\Lambda))/2} \approx \underset{(1127)}{(m(\Xi)+m(N))/2} \tag{3.3}$$

The $\Lambda$ and $\Sigma$ masses are expected to differ, however, because the ace $A_3$ is bound differently in the two cases.

A natural assumption concerning binding energies would be

$$1/2(E^8_{33.}+E^8_{\alpha\beta.}) \approx E^8_{3\alpha.}=E^8_{3\beta.}, \quad 1/2(E^8_{3.3}+E^8_{\alpha.\beta}) \approx E^8_{3.\alpha}=E^8_{3.\beta} \tag{3.4}$$

where $\alpha, \beta=1,2$ [13]. We then obtain in second approximation the familiar

$$\underset{(1127)}{(m(N)+m(\Xi))/2} = \underset{(1134)}{(3m(\Lambda)+m(\Sigma))/4} \quad . \tag{3.5}$$

It is interesting to note that if one assumes that the breaking of unitary symmetry by electromagnetism takes place by virtue of the fact that the $A_2A_1$ mass difference is not zero, then independent of the values of the binding energies we have the mass difference equation [14]:

$$\underset{(6 \pm 1.3)}{m(\Xi^-)-m(\Xi^0)} \approx \underset{(7.0 \pm 0.5)}{m(\Sigma^-)-m(\Sigma^+) - (m(n)-m(p))} \quad . \tag{3.6}$$

Assuming that $A_2$ (the more negative member of the doublet) is heavier than $A_1$ and neglecting shifts in binding energies due to the electromagnetic breaking of the symmetry we find the qualitatively correct result that within any charge multiplet, the more negative the particle, the heavier the mass. Fig. 3 shows the baryon octet after $SU_3$ has been broken by the strong and electromagnetic interactions.

If we further assume in analogy to (3.4) that

$$1/2(E^8_{11.}+E^8_{22.}) \approx E^8_{12.} \quad , \qquad 1/2(E^8_{1.1}+E^8_{2.2}) \approx E^8_{1.2} \tag{3.7}$$

we find

$$\begin{array}{ccc} 1/2(m(\Sigma^+)+m(\Sigma^-)) & = & m(\Sigma^0) \\ (1193.4 \pm 0.3) & & (1193.2\pm 0.7) \end{array} \tag{3.8}$$

B. <u>The Baryon Decuplet</u>

The decuplet states are also constructed from the treys $T_{abc}$. However, the bindings between the aces that form the decuplet may be different from the bindings that were used to form the octet. This is exemplified in Fig. 1b. The difference between octet and decuplet ace bindings is essential, for only in this way are we able to allow for the fact that a $J^P = 1/2^+$ octet of baryons exists while a decuplet with the same spin and parity is absent. The model we are considering is unable to tell us which of the representations 1, 8 or 10, if any, is to be used for a particular baryon spin parity assignment. Nevertheless, once we know of the existence of a baryon singlet, octet, or decuplet, we are able to derive properties of its member states. We will now consider the case of a baryon decuplet as applied to $\Delta_\delta$ (1238), $\Sigma_\delta$ (1385), $\Xi_\delta$ (1530), $\Omega_\delta$(1686). First we list the ten states :

$$
\begin{aligned}
\Delta_\delta^{++} &= T_{111}, = T_{111} \\
\Delta_\delta^{+} &= 1/\sqrt{12}\ T_{112}, = 1/\sqrt{3}\ (T_{112}+T_{121}+T_{211}) \\
\Delta_\delta^{o} &= 1/\sqrt{12}\ T_{221}, = 1/\sqrt{3}\ (T_{221}+T_{212}+T_{122}) \\
\Delta_\delta^{-} &= T_{222}, = T_{222} \\
\Sigma_\delta^{+} &= 1/\sqrt{12}\ T_{113}, = 1/\sqrt{3}\ (T_{113}+T_{131}+T_{311}) \\
\Sigma_\delta^{o} &= 1/\sqrt{6}\ \ T_{123}, = 1/\sqrt{6}\ (T_{123}+T_{231}+T_{312}+T_{132}+T_{321}+T_{213}) \\
\Sigma_\delta^{-} &= 1/\sqrt{12}\ T_{223}, = 1/\sqrt{3}\ (T_{223}+T_{232}+T_{322}) \\
\Xi_\delta^{o} &= 1/\sqrt{12}\ T_{331}, = 1/\sqrt{3}\ (T_{133}+T_{313}+T_{331}) \\
\Xi_\delta^{-} &= 1/\sqrt{12}\ T_{332}, = 1/\sqrt{3}\ (T_{233}+T_{323}+T_{332}) \\
\Omega_\delta^{-} &= T_{333}, = T_{333}
\end{aligned}
\tag{3.9}
$$

In the limit of unitary symmetry $A_1$, $A_2$, and $A_3$ are indistinguishable so that the ten states are completely degenerate. As for the baryon octet, we assume that unitary symmetry is mainly broken by virtue of the fact that $A_3$ is heavier than $A_1$ and $A_2$. The objects of the decuplet will no longer be identical but appear as in Fig. 4. Neglecting shifts in binding energies due to the breaking of unitary symmetry, it is clear that the decuplet resonances increase their masses linearly with strangeness, so that we obtain the analogue of equation (3.3) :

$$
\underset{(147)}{m(\Sigma_\delta)-m(\Delta_\delta)} \approx \underset{(145)}{m(\Xi_\delta)-m(\Sigma_\delta)} \approx \underset{(\sim 153)}{m(\Omega_\delta)-m(\Xi_\delta)} \quad . \tag{3.10}
$$

At first sight it may seem surprising that (3.10) works much better than (3.3) even though both these relations are obtained in the same degree of approximation. However, if we go to the "next order" and assume for the decuplet binding energies $E^{10}$ relations similar to (3.4), i.e.,

$$1/2(E^{10}_{33.}+E^{10}_{\alpha\beta.}) \approx E^{10}_{3\alpha.} = E^{10}_{3\beta.}, \quad 1/2(E^{10}_{3.3}+E^{10}_{\alpha.\beta}) \approx E^{10}_{3.\alpha} = E^{10}_{3.\beta},$$

$$1/2(E^{10}_{.33}+E^{10}_{.\alpha\beta}) \approx E^{10}_{.3\alpha} = E^{10}_{.3\beta}; \quad \alpha,\beta = 1,2 \tag{3.11}$$

we find that the analogue of (3.5) is

$$m(\Sigma_\delta)-m(\Delta_\delta) \approx m(\Xi_\delta)-m(\Sigma_\delta) \approx m(\Omega_\delta)-m(\Xi_\delta)$$

which is just relation (3.10)

There exists one more interesting decuplet mass formula. Independent of the binding energies we find

$$\underset{(\sim 1686)}{m(\Omega_\delta)} = \underset{(1690)}{m(\Delta_\delta)+3(m(\Xi_\delta)-m(\Sigma_\delta))} \quad ^{15)} \quad . \tag{3.12}$$

Since the decuplet and octet are constructed from the same set of particles, we may try to obtain a formula relating the masses of the two different representations. For example,

$$m(\Xi_\delta)-m(\Sigma_\delta) = m(3)-m(\alpha)-1/3(E^{10}_{33.}-E^{10}_{\alpha\beta.}+E^{10}_{3.3}-E^{10}_{\alpha.\beta}+E^{10}_{.33}-E^{10}_{.\alpha\beta}) \tag{3.13}$$

($\alpha,\beta = 1,2$ depending on the charges we take). Now, $m(\Lambda)-m(N)$, $m(\Sigma)-m(N)$, $m(\Xi)-m(\Lambda)$, $m(\Xi)-m(\Sigma)$ all contain the difference $m(3)-m(\alpha)$ and are of roughly the right order of magnitude. Assuming that

$$E^{10}_{33.}-E^{10}_{\alpha\beta.} = E^{10}_{3.3}-E^{10}_{\alpha.\beta} = E^{10}_{.33}-E^{10}_{.\alpha\beta} = E^{8}_{33.}-E^{8}_{\alpha\beta.} = E^{8}_{3.3}-E^{8}_{\alpha.\beta} \tag{3.14}$$

we obtain

$$\underset{(145)}{m(\Xi_\delta)-m(\Sigma_\delta)} \approx \underset{(130)}{m(\Xi)-m(\Sigma)} \quad . \tag{3.15}$$

Note that we do not expect this equation to hold exactly, even in the limit of unitary symmetry: first, because the spins and hence the ace dynamics or binding energies differ for the two representations; second, because octet and decuplet bindings may differ intrinsically, even though the members of the octet and decuplet have the same spins and parities.

Once again assuming that the $A_2A_1$ mass difference accounts for the electromagnetic breaking of unitary symmetry we obtain as counterparts to (3.6) :

$$
\begin{aligned}
m(\Delta_\delta^0)-m(\Delta_\delta^+) &= 1/2(m(\Delta_\delta^-)-m(\Delta_\delta^0)+m(\Delta_\delta^+)-m(\Delta_\delta^{++}))\\
m(\Sigma_\delta^0)-m(\Sigma_\delta^+) &= 1/2(m(\Xi_\delta^-)-m(\Xi_\delta^0)+m(\Delta_\delta^+)-m(\Delta_\delta^{++}))\\
m(\Sigma_\delta^-)-m(\Sigma_\delta^0) &= 1/2(m(\Xi_\delta^-)-m(\Xi_\delta^0)+m(\Delta_\delta^-)-m(\Delta_\delta^0))
\end{aligned}
\qquad (3.16)
$$

Because of the success of relation (3.8) we are tempted to assume

$$
\begin{aligned}
1/2(E^{10}_{11.}+E^{10}_{22.}) &\approx E^{10}_{12.}\\
1/2(E^{10}_{1.1}+E^{10}_{2.2}) &\approx E^{10}_{1.2}\\
1/2(E^{10}_{.11}+E^{10}_{.22}) &\approx E^{10}_{.12}
\end{aligned}
\qquad (3.17)
$$

which immediately yeilds

$$1/2(m(\Sigma_\delta^+)+m(\Sigma_\delta^-)) = m(\Sigma_\delta^0) \quad . \qquad (3.18)$$

Figure 5 shows the baryon decuplet after $SU_3$ has been broken by the strong and electromagnetic interactions.

Relations between electromagnetic mass splittings in the baryon octet and decuplet may be found if we assume the analogues of (3.14), i.e.,

$$E^{10}_{22.}-E^{10}_{11.} \approx E^{10}_{2.2}-E^{10}_{1.1} \approx E^{10}_{.22}-E^{10}_{.11} \approx E^{8}_{22.}-E^{8}_{11.} \approx E^{8}_{2.2}-E^{8}_{1.1}$$

$$E^{10}_{22.}-E^{10}_{12.} \approx E^{10}_{2.2}-E^{10}_{1.2} \approx E^{10}_{.22}-E^{10}_{.12} \approx E^{8}_{22.}-E^{8}_{12.} \approx E^{8}_{2.2}-E^{8}_{1.2} \qquad (3.19)$$

$$E^{10}_{23.}-E^{10}_{13.} \approx E^{10}_{2.3}-E^{10}_{1.3} \approx E^{10}_{.23}-E^{10}_{.13} \approx E^{8}_{23.}-E^{8}_{13.} \approx E^{8}_{2.3}-E^{8}_{1.3} \approx E^{8}_{.23}-E^{8}_{.13}$$

From these follow

$$\underset{(?)}{m(\Delta^{-}_{\delta})-m(\Delta^{++}_{\delta})} \approx \underset{(3.9)}{3(m(n)-m(p))}$$

$$\underset{(17 \pm 7)}{m(\Sigma^{-}_{\delta})-m(\Sigma^{+}_{\delta})}\,^{16)} \approx \underset{(8.25 \pm 0.65)}{m(\Sigma^{-})-m(\Sigma^{+})} \qquad (3.20)$$

$$\underset{(?)}{m(\Xi^{-}_{\delta})-m(\Xi^{0}_{\delta})} \approx \underset{(6.0 \pm 1.3)}{m(\Xi^{-})-m(\Xi^{0})} \quad .$$

Using methods different from ours, Oakes [17] and Rosen [18] have obtained not only the equations (3.16), but also

$$m(\Delta^{-}_{\delta})-m(\Delta^{+}_{\delta}) = \underset{(17 \pm 7)}{m(\Sigma^{-}_{\delta})-m(\Sigma^{+}_{\delta})} \quad . \qquad (3.21)$$

We are unable to obtain this result without the further ad hoc assumptions. In fact, in our model it is natural to expect that within any baryon charge multiplet, the more negative the particle, the heavier it is. Consequently, we might think that for the left-hand side of (3.21),

$$m(\Delta^{-}_{\delta})-m(\Delta^{+}_{\delta}) < m(\Delta^{-}_{\delta})-m(\Delta^{++}_{\delta}) \approx 3(m(n)-m(p)) = 3.9 \qquad (3.22)$$

which is in contradiction with Oakes and Rosen's result (3.21) if we believe in the large $\Sigma_\delta^- \Sigma_\delta^+$ mass difference that is implied by either experiment or by our relation (3.20). Consequently, a measurement of the $\Sigma_\delta^- \Sigma_\delta^+$ mass difference is of some interest.

C. The Baryon Singlet

The $\Lambda_\beta(1405)$, in the limit of unitary symmetry, is shown in Fig. 1c. Figure 6 indicates the $\Lambda_\beta$ when the symmetry is broken by the strong and electromagnetic interactions. Since the $\Lambda_\beta$ is a unitary singlet nothing quantitative can be said about its mass.

## IV. THE VECTOR MESON OCTET AND SINGLET

Meson states are built from the same units $(A_1, A_2, A_3)$ as the baryons. They are contained in the anti-ace × ace cross product space :

$$A^a \times A_b \sim (D^a_b - 1/3\,\delta^a_b D^c_c) + \delta^a_b D^c_c \qquad (4.1)$$
$$\bar{3} \times 3 \sim \quad 8 \quad + \quad 1$$

where $A^a$ stands for the anti-ace of $A_a$. Because of the nature of the decomposition of $\bar{3} \times 3$, mesons can only fall into groups of 8 or 1. The Eightfold Way would allow, in addition, groups of 10, $\overline{10}$, and 27, possibilities which nature does not seem to take advantage of. We have pictorially represented in Figs. 1d, 1e, the two possible meson representations in the limit of unitary symmetry. In order to explain the experimental data it is necessary to assume that the octet and singlet $\bar{A}A$ bindings are the same, at least for the vector meson case.

The vector meson states are given by :

$$\rho^- = D^1_2 \;;\quad \rho^0 = 1/\sqrt{2}(D^1_1 - D^2_2) \;;\quad \rho^+ = D^2_1$$

$$K^{*0} = D^3_2 \;;\quad K^{*+} = D^3_1 \;:\quad K^{*-} = D^1_3 \;:\quad \bar{K}^{*0} = D^2_3 \qquad (4.2)$$

$$\omega_8 = 1/\sqrt{6}(D^1_1 + D^2_2 - 2D^3_3)$$

for the octet, and

$$\omega_0 = 1/\sqrt{3}\; D^c_c = 1/\sqrt{3}(D^1_1 + D^2_2 + D^3_3) \qquad (4.2a)$$

for the unitary singlet. In the limit of unitary symmetry the masses of the singlet and octet must be the same because the binding is identical in both representations and all aces are degenerate. It is important to note that this is not the case for baryons where we have assumed that the singlet, octet, and decuplet bindings all differ, even in the unitary symmetric limit.

Unitary symmetry must be broken for the mesons in exactly the same way as it was broken for the baryons, that is, the isospin singlet $A_3$ (or its anti-ace $A^3$) must become heavier than the isospin doublet $(A_1, A_2)$. Breaking the symmetry by giving $A_3$ a larger mass not only splits the masses of the eight vector mesons, but it also mixes the singlet $\omega_0$ with the $I = 0$ member, $\omega_8$, of the octet. As a result of mixing the physically observable particles $\omega$ and $\varphi$ are formed. Since $A_3$ becomes distinguishable from $A_1$ and $A_2$, $\omega_0$ and $\omega_8$ must mix in such a way as to separate $(A_1, A_2)$ from $A_3$. This immediately leads to

$$\begin{aligned} \varphi &= D_3^3 \\ \omega &= 1/\sqrt{2}(D_1^1 + D_2^2) \quad . \end{aligned} \tag{4.3}$$

The plus and not the minus sign that appears in the "deuce" expression for $\omega$ distinguishes the $\omega$ from the $\rho^0$. Figure 7 shows the vector meson states after unitary symmetry has been broken. Using the empirical fact that when dealing with mesons one must always work with squares of masses, and neglecting changes in the bindings due to the breaking of unitary symmetry, we immediately have [19] :

$$\begin{array}{cc} m^2(\omega) \approx m^2(\rho) \\ (784)^2 \quad (750)^2 \end{array} \tag{4.4} \;{}^{*)}$$

$$\begin{array}{cc} m^2(\varphi) \approx 2m^2(K^*) - m^2(\rho) \quad . \\ (1018)^2 \qquad (1007)^2 \end{array} \tag{4.5} \;{}^{*)}$$

Mixing has made the $\varphi$ as heavy as possible. The mixing angle $\Theta$ defined by

$$\begin{aligned} \varphi &= \omega_0 \mathrm{SIN}\,\Theta - \omega_8 \mathrm{COS}\,\Theta \\ \omega &= \omega_0 \mathrm{COS}\,\Theta + \omega_8 \mathrm{SIN}\,\Theta \end{aligned} \tag{4.6}$$

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*) These relations continue to hold when we include binding energy effects if we assume the meson analogue of (3.4).

comes out to be

$$\mathrm{SIN}\,\Theta = \sqrt{1/3} \quad , \quad \mathrm{COS}\,\Theta = \sqrt{2/3} \quad , \quad \text{or} \quad \Theta \approx 35.3^{\circ} \tag{4.7}$$

as compared with the empirical value of $\Theta \approx 38^{\circ}$ [20].

Only now has the real power of dealing with three basic objects become apparent. When working with the baryons, one could easily say, for example, that the more strangeness a particle carries, the heavier it is. But by using the basic triplet of aces we are able to say, after inspecting the baryons, that for an octet and singlet of mesons it is a non-strange particle that is heaviest of all; for it contains more $A_3$ than the strangeness carrying meson does.

Interestingly enough, we are able to improve equations (4.4) and (4.5). If we define the traceless matrix V of the vector meson octet in the conventional way :

$$V = \begin{pmatrix} \omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}$$

and let the matrix G be given by

$$G_{ab} = V_{ab} + \delta_{ab}\,\omega_0/\sqrt{3} \tag{4.8}$$

then the mass formulae (4.4), (4.5) may be alternatively derived by assuming that

$$H^2 \approx H_1^2 = m_1^2\,\mathrm{Tr}\,\bar{G}G - m_2^2\,\mathrm{Tr}\,\bar{G}(G\lambda_8 + \lambda_8 G) \tag{4.9}$$

for the mass terms in the square of the Hamiltonian H [21]. Here Tr stands for trace while

$$m_1^2 = (2m^2(K^*)+m^2(\rho))/3 \ , \qquad m_2^2 = (m^2(K^*)-m^2(\rho))/3 \ ,$$

and

$$\lambda_8 = 1/\sqrt{3} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} .$$

Note that we have suppressed all terms involving $\mathrm{Tr}\, G = \sqrt{3}\ \omega_0$.

More generally, however, we may write for the mass terms in the square of the Hamiltonian,

$$H^2 = H_1^2 + m_3^2 \ \mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G + m_4^2(\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G\,\lambda_8 + \mathrm{Tr}\,G\,\mathrm{Tr}\,\bar{G}\,\lambda_8) +$$

$$+ m_5^2 \ \mathrm{Tr}\,\bar{G}\,\lambda_8 \mathrm{Tr}\,G\,\lambda_8 + m_6^2 \ \mathrm{Tr}\,\bar{G}\,\lambda_8\, G\lambda_8 \ , \tag{4.10}$$

where we treat the terms in $m_3^2$ to $m_6^2$ as perturbations to $H_1^2$. Since the term $m_3^2 \mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G$ is invariant under $SU_3$ while the terms multiplying $m_4^2$, $m_5^2$, and $m_6^2$ are not, we might expect that to a good approximation we only need keep the perturbation $\mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G$, i.e.,

$$H^2 = H_1^2 + m_3^2 \mathrm{Tr}\,\bar{G}\,\mathrm{Tr}\,G \quad . \tag{4.11}$$

Doing this we immediately arrive at

$$m^2(\omega) - m^2(\rho)/2 \approx m^2(\varphi) + m^2(\rho) - 2m^2(K^*) \tag{4.12}$$

which is correct to the known accuracy of the masses.

## V. THE PSEUDOSCALAR MESONS

Let us assume that in the limit of unitary symmetry we have nine pseudoscalar mesons of equal mass, just like the vector meson case. The members of the octet we call $(\pi, K, \eta_8)$ while the singlet is denoted by $\eta_0$. Breaking the symmetry by increasing the $A_3$ mass yields relations analogous to (4.4) and (4.5), i.e.,

$$m^2(\pi_0^0) \approx m^2(\pi) \tag{5.1}$$

$$\underset{(550)^2}{m^2(\eta)} \approx \underset{(690)^2}{2m^2(K)-m^2(\pi)} \tag{5.2}$$

where $\eta$ and $\pi_0^0$ are the physically observable particles that result from mixing $\eta_8$ and $\eta_0$, just as $\varphi$ and $\omega$ are mixtures of $\omega_8$ and $\omega_0$. Furthermore, by using arguments identical to those given in the vector meson case, we obtain the analogue of the mass relation (4.12)

$$(m^2(\pi_0^0)-m^2(\pi))/2 \approx m^2(\eta)+m^2(\pi)-2m^2(K) \; . \tag{5.3}$$

Substituting the physical masses for $\pi$, K, and $\eta$ we see that $m^2(\pi_0^0)$ comes out negative !

Fortunately, we have an argument that alleviates these obvious difficulties [22]. After increasing the $A_3$ mass we found $m^2(\pi_0^0) \approx m^2(\pi)$. Therefore in this approximation, and this is the crucial point, $m^2(\pi_0^0)$ is very small compared to the mass square differences that exist among the pseudoscalar mesons. A small perturbation (one which changes mass squares by an amount small compared to changes initiated by the $A_3$ mass increase) may be enough to shift the mass square of the $\pi_0^0$ down to zero or even negative values. We might say that the $\pi_0^0$ is formed from two very massive objects that are extremely tightly bound. Energy conservation leaves the $\pi_0^0$ with a small positive energy or mass. If we introduce a perturbation that decreases the mass of the fundamental objects or increases the binding strength then the $\pi_0^0$ may no longer possess a net positive energy and cannot correspond to a physical particle. This, or something like it may be the situation in the pseudoscalar meson case [23].

It is interesting to note that we would not expect the removal of the $\omega$ in analogy to the elimination of the $\pi_0^0$. The perturbation given by (4.11) is expected to shift $m^2(\omega)$ by an amount small compared to the mass square splittings induced by the increase of the $A_3$ mass. Since $m^2(\omega)$ is larger than the vector meson mass square splittings there is no danger of the $\omega$'s disappearing through the introduction of a perturbation.

With the removal of the $\pi_0^0$ we expect that the pseudoscalar mesons behave as an isolated octet. This is indicated in Fig. 8. Neglecting changes in the binding energies due to the breaking of unitary symmetry we immediately obtain, by counting squares, the celebrated Gell-Mann - Okubo formula :

$$m^2(K) \approx 3/4\, m^2(\eta) + 1/4\, m^2(\pi) \qquad (5.4)\ ^{*)}$$

Neglecting differences of binding energies within octets it is clear that we have the relation

$$\begin{array}{ccc} m^2(K^*) - m^2(\rho) & \approx & m^2(K) - m^2(\pi) \\ (0.22\ \mathrm{GeV}^2) & & (0.22\ \mathrm{GeV}^2) \end{array} \qquad (5.5)$$

Note that although it is imperative to assume identical octet and singlet $\bar{A}A$ bindings for the vector mesons, it is by no means clear that a similar situation must exist for other meson representations. It is possible that the pseudoscalar meson singlet $\eta_0$ is not bound and that the binding of $\omega_0$ and the equality of the $\omega_0$ and $\omega_8$ masses in the limit of $SU_3$ symmetry is a very special circumstance, perhaps being due to principles that are not yet fully understood. Indeed, the vector mesons do enjoy a privileged position in that they are coupled to conserved currents.

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*) This relation continues to hold when we include binding energy

If we assume that the $A_2A_1$ mass difference accounts for the electromagnetic breaking of unitary symmetry we are unable to obtain meson mass difference equations that are independent of the binding energies [analogues of (3.6) and (3.16)]. The reason is that there are fewer independent electromagnetic mass differences among the mesons than among the baryons. Furthermore, attempts to generalize binding energy relations like (3.7) or (3.17) to mesons ends in failure $(m^2(\pi^{\pm}) = m^2(\pi^{o}))$, so that we are unable to say anything about meson electromagnetic mass differences. In Figure 9 we display the pseudoscalar mesons after unitary symmetry has been broken by the strong and electromagnetic interactions.

## VI. VECTOR MESON DECAYS

### A. $V \to V+P$

In the ace model the vector - vector - pseudoscalar couplings (VVP) are easily determined pictorially. As an example we indicate in Fig. 10 how the coupling $\langle|\bar{\omega}K^{*+}K^{-}|\rangle$ is obtained. Figure 11 further exemplifies this graphical technique. In this manner we obtain the interaction given in Appendix B. For processes like $K^{*} \to K^{*}+\pi$ or $\rho \to K^{*}+\bar{K}$ we are just determining the $SU_3$ invariant interaction that is conventionally called D type coupling. The Eightfold Way would have given us an identical answer. But unlike the Eightfold Way the ace model also tells us how $\omega$ and $\varphi$ couple. For example,

$$\varphi \to \rho + \pi \quad \text{is forbidden,} \tag{6.1}$$

as indicated in Figure 12. Experimentally, the decay rate for this mode is depressed by at least a factor of 200, as compared with what one might expect on the basis of phase space arguments. It is important to note that the coupling scheme we advocate contains within it information regarding the way unitary symmetry is broken, i.e., $\varphi \to \rho + \pi$ is forbidden after $SU_3$ is violated by the strong interactions: consequently we expect that this decay mode should be greatly inhibited. In fact, (6.1) is the coupling analogue of (4.4) and (4.5). Any theory based on the exactness of unitary symmetry and forbidding $\varphi \to \rho\pi$ would be inadequate for we know the symmetry is too badly broken to be consistent with the experimentally determined suppression of this mode.

Using the G matrix defined in (4.8) and letting

$$P = \begin{pmatrix} \eta/\sqrt{6} + \pi^{0}/\sqrt{2} & \pi^{+} & K^{+} \\ \pi^{-} & \eta/\sqrt{6} - \pi^{0}/\sqrt{2} & K^{0} \\ K^{-} & \bar{K}^{0} & -2\eta/\sqrt{6} \end{pmatrix} \tag{6.2}$$

one can easily show that the above coupling scheme is equivalent to letting the interaction Hamiltonian $H_{int}$ be given by

$$H_{int} \propto \mathrm{Tr}\,\overline{G}\,(GP+PG) \quad . \qquad (6.3)$$

Note that terms proportional to $\mathrm{Tr}\,G = \sqrt{3}\,\omega_0$ are not included [21]. We have found a similar situation when using the matrix formulation to derive $m^2(\omega) \approx m^2(\rho)$.

The $\rho$, $K^*$, $\omega$ and $\varphi$ states given in (4.2) and (4.3) are eigenstates of the Hamiltonian $H_1$ displayed in (4.9). It is interesting to ask for the eigenstates of the perturbed system determined by (4.11), i.e., for

$$H^2 = H_1^2 + m_3^2\,\mathrm{Tr}\,\overline{G}\,\mathrm{Tr}\,G = H_1^2 + m_3^2(2\bar{\omega}\omega + \bar{\varphi}\varphi + \sqrt{2}\,\bar{\omega}\varphi + \sqrt{2}\,\bar{\varphi}\omega) \quad . \qquad (4.11)$$

Only $\varphi = D_3^3$ and $\omega = 1/\sqrt{2}(D_1^1 + D_2^2)$ are affected by the perturbation. The perturbed eigenstates $\omega'$, and $\varphi'$ are given by

$$\begin{aligned} \varphi' &= \mathrm{COS}\beta\,\varphi + \mathrm{SIN}\beta\,\omega \approx \varphi + \beta\,\omega \\ \omega' &= \mathrm{COS}\beta\,\omega - \mathrm{SIN}\beta\,\varphi \approx \omega - \beta\,\varphi \end{aligned} \qquad (6.4)$$

where

$$\beta = \sqrt{2}\,m_3^2/(m^2(\varphi) - m^2(\omega)) = (m^2(\omega) - m^2(\rho))/(\sqrt{2}(m^2(\varphi) - m^2(\omega))) \quad .$$

The $\varphi$ piece of $\varphi'$ is unable to decay into $\rho + \pi$, as demonstrated in Fig. 12. But the addition of the term $\beta\omega$ allows $\varphi'$ to go to $\rho + \pi$ just as $\omega \to \rho + \pi$ (virtual) $\to \pi + \pi + \pi$. The $\omega\rho\pi$ coupling constant may be estimated from $\Gamma(\omega \to 3\pi) = 8.5$ MeV. We find that if the $\varphi' \to \rho\pi$ coupling were the same as the $\omega \to \rho\pi$ coupling, $\Gamma(\varphi' \to \rho\pi$, all charge states) would equal 54 MeV [24]. Experimentally, $\Gamma(\varphi' \to \rho\pi) \lesssim 1.0 \pm 0.6$ MeV [25]. The model we are considering yields as a crude estimate

$$\Gamma(\varphi' \to \rho\pi, \text{ all charge states}) = \beta^2\ 54\ \text{MeV} \approx 0.3 \pm 0.11\ \text{MeV} \qquad (6.5)$$

for $m(\omega) = 784 \pm 0.9$, $m(\rho) = 754 \pm 5$, or

$$\Gamma(\varphi' \to \rho\pi, \text{ all charge states})/\Gamma(\varphi' \text{ total}) \approx$$
$$\approx 0.3 \pm 0.11/3.1 \pm 1 \sim 0.2 \to 0.04 \qquad (6.6)$$

This is just at the verge of experimental detectibility. The quantity $\beta^2$ is very sensitive to the $\omega\ \rho$ mass difference which accounts for the large errors given in (6.5). To summarize, we have found that $\varphi \to \rho\pi$ is forbidden to the order in which $m^2(\omega) = m^2(\rho)$. The interaction responsible for the splitting of the $\omega\ \rho$ masses also induces the decay $\varphi \to \rho\pi$ with a strength proportional to

$$\left[(m^2(\omega)-m^2(\rho))/m^2(\varphi)-m^2(\omega))\right]^2 .$$

B. V → P+P

Couplings for decays of this type may be computed with pictorial techniques similar to those used for V → V+P. There is one essential difference, however. When forming $\langle\ |\bar{V}VP|\ \rangle$ we allowed V to act on $\bar{V}$ as exemplified in Figs. 10a, 10b, and 10c. In Fig. 10b we see that an open and closed circle at the bottom of two deuces annihilate, which effectively multiplies the coupling by +1. Had this annihilation taken place between objects at the top of two deuces we would also have multiplied our coupling by +1. A situation of this type is considered in Fig. 11. Now, it is perfectly consistent with $SU_3$ to use a +1 for bottom but −1 for top annihilations. This latter approach gives rise to what is called F type couplings. Whether we use F or D coupling for a particular meson interaction is determined by charge conjugation invariance. For VVP we are forced to D type, while for VPP we must use F.

The $\omega - \varphi$ mixing that is implied by our model leads to the VPP couplings given in Appendix B. In matrix notation

$$H_{int} \propto \mathrm{Tr}\,\widetilde{G}_{\mu}(\partial_{\mu}PP-P\,\partial_{\mu}P) \quad . \tag{6.7}$$

We may readily compute $\Gamma(\varphi \rightarrow K+\overline{K})$ in terms of $\Gamma(\rho \rightarrow \pi+\pi)$. For a $\rho$ width of 100 MeV,

$$(\varphi \rightarrow \overline{K}K, \text{ all charge states}) \approx 2 \text{ MeV} , \tag{6.8}$$

as compared with the experimental value [26] of $3.1 \pm 1$ MeV.

## VII. BARYON COUPLINGS

The baryons (mesons) are constructed from linear combinations of treys (deuces). If we understand how to couple two treys $T^{abc}$, $T_{def}$ to a deuce $D^g_h$ we will in effect know how to determine the baryon-baryon-meson couplings (the upper indices on $T^{abc}$ indicate that it is constructed from anti-aces). Two natural ways of proceeding are depicted in Fig. 13. We place the triangle representing the trey $T^{abc}$ next to the triangle representing $T_{def}$ in such a way that corresponding sides match up. Then we see if the two triangles can annihilate with the help of the "dumb-bell" that represents the deuce. Only aces and anti-aces lying on corresponding vertices of the triangles are allowed to annihilate one another, with or without the help of the ace and anti-ace which lie on the ends of the dumb-bell. For example, $T^{abc}T_{abd}D^d_c$ ($c \neq a,b$; $d \neq a,b$; $a \neq b$; no summation over repeated indices unless otherwise specified) gives $+1$,

$$T^{bac}T_{abd}D^d_c = 0, \quad T^{acb}T_{abd}D^d_c = 0, \quad T^{cab}T_{abd}D^d_c = 0, \quad \text{etc.;}$$

$$T^{acb}T_{adb}D^d_c = +1, \quad T^{bca}T_{adb}D^d_c = 0, \quad T^{abc}T_{adb}D^d_c = 0, \quad \text{etc.;}$$

$$T^{cab}T_{dab}D^d_c = 1, \quad T^{cba}T_{dab}D^d_c = 0, \quad T^{bac}T_{dab}D^d_c = 0, \quad \text{etc.}$$

For cases like $T^{acc}T_{acc}D^c_c$ $(a \neq c)$ we count each physically distinct annihilation configuration. Hence, $T^{acc}T_{acc}D^c_c = 2$, $T^{ccc}T_{ccc}D^c_c = 3$, etc. As an example, let us now compute the $\bar{p}n\rho^+$ coupling. With the help of (2.2) and (4.2) :

$$\begin{aligned}
\langle|\bar{p}n\,\rho^+|\rangle &= 1/2(T^{112}-T^{121})(T_{212}-T_{221})D^2_1 \\
&= 1/2(T^{112}T_{212}-T^{112}T_{221}-T^{121}T_{212}+T^{121}T_{221})D^2_1 \\
&= 1/2(T^{112}T_{212}+T^{121}T_{221})D^2_1 \\
&= 1/2 + 1/2 = 1 \quad .
\end{aligned}$$

For $\Xi^0 \, \Xi^- \, \rho^+$,

$$\langle |\Xi^0 \Xi^- \rho^+| \rangle = 1/2(T^{331}-T^{313})(T_{323}-T_{332})D_1^2$$
$$=-1/2(T^{331}T_{332}+T^{313}T_{323})D_1^2$$
$$=-1/2 - 1/2 = -1 \quad .$$

In this way we generate a coupling scheme which is conventionally called F type. There are, however, other possibilities. Let us label the vertices of the trey triangles in clockwise order by 1, 2, 3. We may then multiply the coupling by +1 or −1 depending on the particular vertex where the dumb-bell or deuce acts, rather than our more restrictive previous choice of +1 only. Most "natural" (which means that it is the first possibility we try) is to assign +1 to odd, −1 to even numbered vertices, i.e.,

$$T^{abc}T_{abd}D_c^d = +1, \quad T^{acb}T_{adb}D_c^d = -1, \quad T^{cab}T_{dab}D_c^d = +1, \quad \text{etc.}$$

Using this coupling scheme, labelled $(+-+)$ for obvious reasons, we obtain the "F+D" interaction rather than the pure F of $(+++)$. The other choices $(+--)$ and $(++-)$ yield D and F+D respectively. More complicated ways of obtaining $SU_3$ invariant interactions are discussed in Appendix C.

It is important to note that in discussing the various trey-trey-deuce interactions we are determining not only baryon octet-baryon octet-meson interactions, but also baryon decuplet-baryon octet-meson, baryon singlet-baryon octet-meson, etc., interactions. Table 2 indicates various baryon-baryon-meson couplings that are induced by the coupling types we have discussed. The entries labelled S and T stand for singlet and decuplet interactions given in Appendix C. We shall concentrate primarily on the baryon-baryon-pseudoscalar meson interaction because of the wealth of available experimental information.

It is possible that nature uses a complicated linear combination of the coupling schemes we have mentioned. The F/D ratio would then be determinable only by experiment or dynamical calculations. However, if we require that the coupling be algebraically natural and of the simplest form, we are automatically led to (+−+) or (++−), that is F+D. Any other choice would not generate interactions in all representations. This assumption of simplicity is consistent with all known experimental information and seems, in fact, to be required by existing data. This is discussed in Section IX.

## VIII. THE WEAK INTERACTIONS

We have obtained the result that $\Lambda_o$ is heavier than $(p_o, n_o)$ by an amount characteristic of the gross mass splittings within an octet or decuplet, i.e., $\sim 150$ MeV. We therefore expect that $\Lambda_o$, if it exists, would undergo the $\beta$-decays

$$\begin{aligned} \Lambda_o^{-1/3} &\rightarrow p_o^{+2/3} + e^- + \nu \\ &\rightarrow p_o^{+2/3} + \mu^- + \nu \end{aligned} \tag{8.1}$$

just as

$$\begin{aligned} \Lambda &\rightarrow p + e^- + \nu \\ &\rightarrow p + \mu^- + \nu \end{aligned} \tag{8.2}$$

On the basis of the electromagnetic mass splittings within a given isotopic spin multiplet, we are also tempted to conjecture that $n_o$ is heavier than $p_o$, making $p_o$ completely stable (like $p$) but allowing the decay

$$n_o^{-1/3} \rightarrow p_o^{+2/3} + e^- + \nu \tag{8.3}$$

just as

$$n \rightarrow p + e^- + \nu \tag{8.4}$$

The $\Lambda$ has been considered a bound state of $p_o$, $n_o$, and $\Lambda_o$. Consequently, it is natural to assume that reaction (8.2) takes place as a result of (8.1). Both of these decays are then governed by the same coupling constant. Since the $\Lambda p$ and $\Lambda_o p_o$ mass differences are comparable, we would expect that the $\Lambda$ and $\Lambda_o$ $\beta$-decay lifetimes would be of the same order of magnitude. Similarly, the lifetime of the $n_o$ would be roughly that of the $n$.

A theory of leptonic decays based on the fundamental reactions (8.1) and (8.3) is in fact quite pleasing. We assume that the weak decays of strongly interacting particles are induced by the weak decays of the aces which comprise them. The couplings (weak interaction Lagrangian) may be simply determined graphically. For example, we assume that a baryon undergoes $\beta$-decay when an ace at one of the vertices of the triangle representing the baryon decays into another ace + $e\nu$, while the aces at the other two vertices remain undisturbed. [We immediately see that this is just the coupling type (+ + +) discussed in Section VII, with the meson replaced by $e\nu$.] From this follows :

1) the conserved vector current theory for non-strangeness changing ($\Delta S = 0$) leptonic decays;

2) the $|\Delta I| = 1/2$, $\Delta S/\Delta Q = +1$ rules for $|\Delta S| = 1$ leptonic decays;

3) $$\Omega_\delta^- \rightarrow \Xi^0 + e^- + \nu \quad \text{is forbidden} \tag{8.5}$$

if we demand that the same coupling type be used for the decuplet as for the octet of baryons [see Table 2, entry (+ + +)]. However, the reaction

$$\Omega_\delta^- \rightarrow \Xi_\delta^0 + e^- + \nu \quad \text{is allowed.} \tag{8.6}$$

In analogy with previous work [27] we assume that the space-time part of the weak interactions is to be written in terms of right and left handed aces, i.e., a so-called $1 \pm \gamma_5$ [28] theory. Departures from this type of interaction are attributed to the breaking of unitary symmetry. Unfortunately, we are unable to estimate quantitatively how badly the symmetry is violated. However, neutron $\beta$-decay affords us a hint in this direction. In the unitary symmetric limit we obtain for the interaction Hamiltonian :

$$H_{int} \propto \bar{n}\, \gamma_\mu (1 \pm \gamma_5) p\, \bar{\nu}\, \gamma_\mu (1 + \gamma_5) e \tag{8.7}$$

where there is an uncertainty as to whether the + or - sign is correct (in this connection see Ref. 27)). Experimentally,

$$H_{int} \propto \bar{n}\, \gamma_\mu (1+1.25\, \gamma_5) p\, \bar{\nu}\, \gamma_\mu (1+ \gamma_5) e \tag{8.8}$$

If the + sign is to be used in (8.7) then there is a chance that the symmetry remains recognizable after it is violated. Table 3 contains predictions of the theory for $|\Delta S| = 1$ $\beta$-decays. The results are compared with experiment and the work of Cabibbo 29). Note that we have also included numbers for $\Sigma^- \to \Lambda + e^- + \nu$. The conserved vector current theory predicts that the vector part of this decay vanishes. We have assumed, in addition, that vector and axial vector should always enter in equal strength. Hence, our model forbids this decay. To obtain some feeling for what forbidden means, we have assumed that unitary symmetry breaking interactions have changed $\bar{\Sigma}^- \gamma_\mu (0+0\,\gamma_5)\Lambda$ to $\bar{\Sigma}^- \gamma_\mu (0+0.25\,\gamma_5)\Lambda$ much like $\bar{n}\, \gamma_\mu (1+ \gamma_5) p$ becomes $\bar{n}\, \gamma_\mu (1+1.25\, \gamma_5) p$. We then obtain a branching ratio of $\Gamma(\Sigma^- \to \Lambda + e^- + \nu)/\Gamma(\Sigma^- \text{ total}) \approx$ $\approx 10^{-5}$, which is compatible with the still very crude (9 events) experimental result.

The ace $\Lambda_o$ is also expected to undergo non-leptonic decays :

$$\begin{aligned} \Lambda_o^{-1/3} &\to p_o^{+2/3} + \pi^- \\ \Lambda_o^{-1/3} &\to n_o^{-1/3} + \pi^o \end{aligned} \tag{8.9}$$

If these reactions obey a $|\Delta I| = 1/2$ non-leptonic decay rule, then so will the baryon and meson non-leptonic decays. The rates for (8.9) are comparable to the rates for $\Lambda$ non-leptonic decays since similar mass differences and coupling constants are used in the two cases.

It is possible that the $p_o$ and $\Lambda_o$ lifetimes are not primarily determined by (8.1), (8.3), and (8.9). For example, if the $\overline{AA}$ system binds we might have

$$\begin{aligned} &\Lambda_o^{-1/3} \rightarrow \Lambda + (\bar{p}_o^{-2/3}\ \bar{n}_o^{+1/3}) \\ &n_o^{-1/3} \rightarrow n + (\bar{p}_o^{-2/3}\ \bar{n}_o^{+1/3}) \quad . \end{aligned} \tag{8.10}$$

Both these reactions could proceed via the strong interactions and hence make $\Lambda_o$ and $n_o$ short-lived "resonances". Similarly for $p_o$.

However, a crude estimate of the $\overline{AA}$ mass indicates that (8.10) is energetically impossible. We argue as follows :

$$m(\overline{AA}) \approx 2m(\bar{A}) - E(\overline{AA}) \tag{8.11}$$

where $m(\overline{AA})$ is the mass of $\overline{AA}$ and $E(\overline{AA})$ is the $\overline{AA}$ binding energy. Since

$$\begin{aligned} m(\text{antibaryon}) \equiv m(\bar{B}) &\approx 3m(\bar{A}) - 3E(\overline{AA}) \quad , \\ m(\overline{AA}) &\approx m(\bar{A}) + \frac{m(\bar{B})}{3} \quad , \end{aligned} \tag{8.12}$$

forbidding (8.10).

## IX. OTHER BARYONIC STATES

On the basis of the results obtained in this paper we would like to suggest the possible existence of a $J^P = 3/2^-$ baryon octet ($\gamma$) and a $J^P = 3/2^+$ baryon singlet ($\delta$) containing the following members :

$$N_\gamma(1515), \quad \Lambda_\gamma(1635)?, \quad \Sigma_\gamma(1660), \quad \Xi_\gamma(1770)^{30)} \quad \text{(octet);}$$

$$\Lambda_\delta(1520) \qquad \text{(singlet).} \tag{9.1}$$

The $\Lambda_\gamma(1635)$ has yet to be discovered, although its effects may already have been observed.

This is to be compared with the Glashow-Rosenfeld [31] assignments of

$$N_\gamma(1515), \quad \Lambda_\gamma(1520), \quad \Sigma_\gamma(1660), \quad \Xi_\gamma(1600)? \; . \tag{9.2}$$

Note that the mass structure of the Glashow-Rosenfeld $\gamma$ - octet is untenable from our point of view for the mass differences bear no resemblance to those of the $N, \Lambda, \Sigma, \Xi$. The $\Xi_\gamma(1600)$ has been looked for but has not yet been found. Furthermore, there are indications that the parity of the $\Lambda(1520)$ is opposite to that of the $N(1515)$ [32].

Using the F+D coupling, as indicated in Section VII, we have found partial widths for the $\gamma$ - octet. Results for the $\gamma$ - octet and $\delta$ - singlet are shown in Table 4 and are in satisfactory agreement with experiment. Since the $\pi\Sigma$ decay mode of the $\Lambda_\gamma$ is not dominant, it is not unreasonable to suppose that it has been missed in experiments looking for $\pi\Sigma$ enhancements [33),34)].

Because $m(\Lambda_\gamma) \approx m(\Sigma_\gamma)$, there is the danger of confusing the two resonances. Bastien and Berge [34] have made a careful study of $K^-p$ elastic scattering in a region where the total energy in the centre-of-mass system is $\approx 1660$ MeV. They find

$$\Gamma(\Sigma_\gamma \to \bar{K}N) \gtrsim 4 \text{ MeV} \quad . \tag{9.3}$$

Unfortunately, the effects they attribute to the $\Sigma_\gamma$ may in part be due to the $\Lambda_\gamma$ . This experiment alone cannot disentangle the $I = 0$ and $I = 1$ channels. The $I = 1$ channel has been investigated by Alvarez et al. [35] in the reaction $K^- + p \to \pi^- + \bar{K}^0 + p$. They report that, within the statistics based on their 1223 events, no $\bar{K}^0 p$ enhancement is observed. They estimate

$$\Gamma(\Sigma_\gamma \to \bar{K}N) \lesssim 2 \text{ MeV} \tag{9.4}$$

which is consistent with our prediction

$$\Gamma(\Sigma_\gamma \to \bar{K}N) \approx 0 \quad . \tag{9.5}$$

Relations (9.4) and (9.5) are remarkable in that phase space considerations alone would predict much larger widths. If Bastien and Berge are really seeing the combined effects of $\Lambda_\gamma$ and $\Sigma_\gamma$ then (9.3) and (9.4) need not be in contradiction.

We shall follow Glashow and Rosenfeld in assuming the existence of an $\alpha$ - baryon octet composed of

$$N_\alpha(1685), \; \Lambda_\alpha(1815), \; \Sigma_\alpha(1875)?, \; \Xi_\alpha(1972)?. \quad \text{(octet)} \tag{9.6}$$

Once again we use F+D couplings to find the partial widths for the decay of this octet, as summarized in Table 5. In order to obtain some idea of the expected accuracy of these results we have included in Table 5 theoretical and experimental decay widths of the well established $\delta$ - decuplet [36].

## X. OTHER MESONIC STATES

We consider it an open question as to whether meson octets and singlets must occur together [see Section V]. We proceed with this in mind.

Although the $K$ (725) may be accommodated into a unitary symmetry scheme without any partners [37], it is nevertheless interesting to assume that it is formed from $\bar{A}A$ and to apply therefore equations (5.4), (5.5) to see where its companions lie, if they exist. We find that the $I = 1$ state $\pi^*$ is at ~550, while the $I = 0$ resonance $\eta^*$ appears at ~775 MeV, within the rather broad $\rho$ mass region. It is well known that an asymmetry exists in $\rho^0$ decay [38], while none is present in the decay of charged $\rho$'s. This asymmetry has been associated with a rapidly rising phase shift in the $I = 0$, $J = 0$ or $2$ $\pi\pi$ system. This may in fact be the $\eta^*$. To allow $\eta^* \to \pi^+\pi^-$ we want $G$ parity $+1$. Spin zero is the simplest choice and would also forbid the decay $K^*(888) \to K(725) + \pi$ which experimentally seems somewhat suppressed [39]. Hence, we take for $\eta^*$ $J^{PG} = 0^{++}$ or $0^{-+}$. If we pick $0^{-+}$ then its partner $\pi^*$ would be $0^{--}$ with the principle decay mode being $3\pi$'s. It then should have been seen in some of the experiments that established the existence and quantum numbers of the $\eta$ (550). Since nothing peculiar has been reported we are finally led to a tentative $0^{+-}$ assignment for the $\pi^*$ and $0^{++}$ for the $\eta^*$. The smallest number of $\pi$'s the $\pi^*$ could strongly decay into would then be 5 (3 is excluded by parity), but energy conservation removes this possibility. The principal decay modes of the $\pi^*$ would then be $2\pi + \gamma$ (order $\alpha$) or $2\pi$ (order $\alpha^2$). Like the $K$ it should be produced with a rather small cross-section. It has probably not yet been seen although there is a chance that it might be the $\zeta$. The $\eta^*$ is detected most easily through its mode $\eta^* \to \pi^0 + \pi^0$, since $\rho^0 \to \pi^0 + \pi^0$ is forbidden.

With the help of unitary symmetry we find that

$$\Gamma(\eta^* \text{ total})/\Gamma(K \text{ total}) \approx \Gamma(\eta^* \to \pi\pi)/\Gamma(K \to K\pi) = 1.15 \quad (10.1)$$

Since experimentally $\Gamma(K \text{ total}) < 15$ MeV, $\Gamma(\eta^* \text{ total}) < 17$ MeV.

It is tempting to put the B(1220)[40] and the $K^*\bar{K}$ or $\bar{K}^*K(1410)$[41] bump into the same $SU_3$ representation. Existing data favours a $1^-$ $J^P$ assignment for the B[42], although the evidence is not conclusive. The B and the $f^0$ would then be considered decay modes of the same particle[43]. Let us assume that $J^P = 1^-$ is correct. In order to facilitate the labelling of these mesons we adopt the convention of Rosenfeld, Chew and Gell-Mann. The B would then be called $\pi_\gamma$ while $\varphi_\gamma$ or $\eta_\gamma$ would stand for the bump at 1410 depending upon whether or not there is singlet-octet mixing. We use $\eta_\gamma$ if there is only an octet of mesons. The two possibilities are

$$\pi_\gamma(1220), \qquad K_\gamma(1365)?, \qquad \eta_\gamma(1410) \qquad \text{(octet)} \tag{10.2a}$$

$$\pi_\gamma(1220), \qquad \omega_\gamma(1220)?, \qquad K_\gamma(1320)?, \qquad \varphi_\gamma(1410) \qquad \text{(octet+singlet)} \tag{10.2b}$$

The masses have been obtained with the aid of equations (5.4) for (10.1): (4.4) and (4.5) for (10.2). Note that the ordering of the masses is just as we would expect. Moreover, the (octet+singlet) case has the remarkable property

$$\begin{array}{ccccc} m^2(K_\gamma)-m^2(\pi_\gamma) & \approx & m^2(K^*)-m^2(\rho) & \approx & m^2(K)-m^2(\pi) \\ (\sim 0.24 \ \text{GeV}^2) & & (0.22 \ \text{GeV}^2) & & (0.22 \ \text{GeV}^2) \end{array} \tag{10.3}$$

which is good to known accuracy of the masses [see Equation (5.5)].

The $\gamma$-mesons have two different types of decay channels open to them, as is exemplified by the $\pi_\gamma$ which decays into $\pi\pi$(PP) and $\omega\pi$(VP). Both the PP and VP modes are important, complicating any study of the $\gamma$-meson widths. Furthermore, there is the uncertainty of how to treat the $\omega_\gamma - \varphi_\gamma$ mixing. Fortunately the ace theory suggests answers to these questions. First, $\omega_\gamma - \varphi_\gamma$ mixing must be the same as $\omega - \varphi$ mixing. Second, all couplings may be determined as described in Section VI. Briefly in matrix notation,

$$H_{int} \propto \mathrm{Tr}\,\bar{V}_{\gamma}(PG+GP) \quad \text{and} \quad \mathrm{Tr}\,\bar{V}_{\gamma}PP \qquad \text{(octet)} \tag{10.4}$$

$$H_{int} \propto \mathrm{Tr}\,\bar{G}_{\gamma}(PG+GP) \quad \text{and} \quad \mathrm{Tr}\,\bar{G}_{\gamma}PP \qquad \text{(octet+singlet)} \tag{10.5}$$

where $V_{\gamma}$ and $G_{\gamma}$ are the $\gamma$- meson counterparts of the $V$ and $G$ matrices given in (4.8). The space-time part of the interaction has been suppressed. Finally, we might expect that

$$\Gamma(\pi_{\gamma} \to \omega\pi)/\Gamma(\pi_{\gamma} \to \pi\pi) \approx 1/2 \ . \tag{10.6}$$

This latter result is obtained in the following somewhat indirect manner.

The Gell-Mann - Okubo mass formula may be written for mesons as :

$$m^2 = m_o^2(1+b'(m_o^2)(I(I+1)-1/4\,Y^2))$$

where $m_o$ and $b'$ vary from one representation to another. $b'$ may be considered a function of $m_o^2$, for $m_o^2$ may be taken to label the represent-ations. Equations (5.5) and (10.3) imply that

$$b'(m_o^2) \sim 1/m_o^2 \quad . \tag{10.7}$$

This observation tempts us to assume that if we have two different meson representations of the same spin-parity decaying into similar final states, the couplings $\gamma(m_o^2)$ governing these decays have for their $m_o^2$ dependence :

$$\gamma(m_o^2) \sim 1/m_o^2 \quad . \tag{10.8}$$

This would imply a connection between

a) $\pi_{\gamma} \to \pi\pi$ and $\rho \to \pi\pi$;

b) $\pi_{\gamma} \to \omega\,\pi$ and $\omega \to \rho\pi \to \pi+\pi+\pi.$

We obtain in this manner :

a)

$$\Gamma(\pi_\gamma \rightarrow \pi\pi, \text{ all charge states}) =$$

$$= \left(\frac{594}{348}\right)^3 \left(\frac{750}{1220}\right)^2 \left(\frac{925}{1350}\right)^2 \Gamma(\rho \rightarrow \pi\pi) = 88 \text{ MeV} \tag{10.9}$$

where 594, 348 are $\pi$ momenta; 750, 1220 are $\rho$ and $\pi_\gamma$ masses; 925, 1350 are values of $m_o$ [44].

b)

$$\Gamma(\pi_\gamma \rightarrow \omega\pi, \text{ all charge states}) = 43 \text{ MeV} \tag{10.10}$$

where the $\omega\rho\pi$ coupling has been estimated from the $3\pi$ decay of the $\omega$. This yields

$$\Gamma(\pi_\gamma \text{ total}) \approx 130 \text{ MeV} \qquad \text{(theory)}$$

as compared to (10.11)

$$\Gamma(\pi_\gamma \text{ total}) = 100 \pm 20 \text{ MeV} \qquad \text{(experiment)}$$

and a fortiori equation (10.6).

To obtain some feeling for the partial widths of the $\gamma$- mesons we consider three cases. First we assume that only the PP or VP channels are important. Then we take the case where :

$$\Gamma(\pi_\gamma \rightarrow \omega\pi, \text{ all charge states})/\Gamma(\pi_\gamma \rightarrow \pi\pi, \text{ all charge states}) = 1/2.$$

These results are summarized in Table 6a,b.

We have also included partial widths for the mesons under discussion assuming them to be $J^P = 1^+(\delta)$ instead of the more probable $1^-(\gamma)$. There is satisfactory agreement with experiment only when the mesons form a nonet.

## XI. GENERAL COMMENTS

The degree to which unitary symmetry is violated seems precarious; it appears to change from one representation to another. For the pseudoscalar mesons, for example, the violation seems enormous. Unitary symmetry gives $m^2(\pi) = m^2(K) = m^2(\eta)$, yet, for physical particles $m^2(\pi) << << m^2(K) \approx m^2(\eta)$. For the baryons, on the other hand, unitary symmetry works reasonably well, predicting $m(N) = m(\Lambda) = m(\Sigma) = m(\Xi)$. In spite of these differences, our model suggests that the strength of unitary symmetry violation is the same in both cases; for the breaking of unitary symmetry is measured by ace mass splittings, i.e.,

$$(m(A_3)-m(A_1))/m(A_1)$$

and not by $(m^2(K)-m^2(\pi))/m^2(\pi)$ or $(m(\Lambda)-m(N))/m(N)$. The amount of unitary symmetry breaking is universal, it is the same for mesons as baryons, it is identical for octets and decuplets. This accounts for roughly the same mass differences within the meson octets, the baryon octet, and the baryon decuplet, irrespective of the masses of the members of these representations.

Although our aces $p_o, n_o, \Lambda_o$ have "peculiar" baryon number and charge, their space-time properties should be identical to $p$, $n$, $\Lambda$ (in this respect we may think of them as $p$, $n$, $\Lambda$ with charge translated by a unit of $-1/3$). This places a restriction on the quantum numbers that a meson may possess. For example, for spin 0 or spin 1 non-strange mesons, the following $J^{PG}$ are excluded :

1) $0^{+-}$, $0^{--}$, $1^{-+}$ for isospin 0 states:

2) $0^{++}$, $0^{-+}$, $1^{--}$ for isospin 1 states.

Up to now no resonances have been found with these quantum numbers. Table 7 lists the low mass meson states that may be formed from $\bar{A}A$.

It is natural to associate the baryons with the lowest energy state of the trey system that represents them. This presumably means that the 3 aces are all in orbital angular momentum S states with the spin of one pair summing to 0. Similarly, the pseudoscalar mesons would correspond to an ace and anti-ace whose orbital angular momentum and total spin are both 0 (i.e., $^1S_0$ state). Since the parity of a nucleon (ace) and anti-nucleon (anti-ace) state are opposite, we see that the intrinsic parity of the pion should be odd while that of the nucleon should be even.

If aces exist, they most probably interact strongly, like the nucleon or pion. There are other possibilities, however, which must be kept in mind when designing experiments to detect the aces. For example, there may be an interaction, stronger than the strong interactions, which governs the behaviour of aces causing them to bind to form mesons and baryons. In this model the strong interactions would be viewed as "some kind of van der Waals' force". Just as two isolated electrons do not interact with a van der Waals' force, so two aces do not interact strongly. The ace-like structure of a system would then be discernible at distances measured in terms of the masses of the particles which bind the aces (not the pion mass). Consequently high momentum transfer experiments may be necessary to detect aces.

## XII. CONCLUSIONS

The scheme we have outlined has given, in addition to what we already know from the Eightfold Way, a rather loose but unified structure to the mesons and baryons. In view of the extremely crude manner in which we have approached the problem, the results we have obtained seem somewhat miraculous.

A universality principle for the breaking of unitary symmetry by the strong interactions has been suggested. From this followed a qualitative understanding of the meson mass splittings in terms of the baryon mass spectrum, e.g., $m(\Lambda) > m(N)$ implies that $m(\varphi) > m(K^*) > m(\omega) \approx \approx m(\rho)$. The proportionately larger mass splittings within the pseudoscalar meson octet have been explained. Mass formulae relating members of different representations have been suggested, e.g.,

$$(m^2(\omega)-m^2(\rho))/2 \approx m^2(\varphi)+m^2(\rho)-2m^2(K^*) \quad ,$$

$$m^2(K^*)-m^2(\rho) \approx m^2(K)-m^2(\pi), \quad m(\Xi)-m(\Sigma) \approx m(\Xi_\delta)-m(\Sigma_\delta), \quad \text{etc.}$$

A universality principle for the breaking of unitary symmetry by the electromagnetic interactions has also been assumed. This has led to the qualitatively correct result that within any baryon charge multiplet, the more negative the particle, the heavier the mass. Electromagnetic mass splitting formulae relating members of different representations have been suggested, e.g.,

$$m(\Sigma^+)-m(\Sigma^-) \approx m(\Sigma_\delta^+)-m(\Sigma_\delta^-), \quad m(\Xi^-)-m(\Xi^0) \approx m(\Xi_\delta^-)-m(\Xi_\delta^0), \quad \text{etc.}$$

Nature's seeming choice of 1, 8, and 10-dimensional representations for baryons along with 1 and 8-dimensional representations for the mesons has been accounted for without dynamical or "bootstrap" considerations. The amount of octet-singlet $(\omega - \varphi)$ mixing has also been predicted with algebraic techniques.

A pictorial method for determining strong interaction coupling constants has been presented. A unique baryon-baryon-pseudoscalar meson coupling has been suggested (F+D). We have found that $\varphi \to \rho\pi$ is forbidden to the order in which $m^2(\omega) = m^2(\rho)$. The interaction responsible for the splitting of the $\omega$ $\rho$ masses has induced the decay $\varphi \to \rho\pi$ with a strength proportional to

$$\left[(m^2(\omega)-m^2(\rho))/(m^2(\varphi)-m^2(\omega))\right]^2 .$$

The quantum numbers available to a meson have been restricted to those which may be formed from the p, n, $\Lambda$ and their antiparticles. The odd intrinsic parity of the pion and opposite nucleon parity fit naturally into the model.

The theory has been quantitatively applied to resonances that have not as yet been definitively classified into representations of $SU_3$. $\Lambda_\gamma(1635)$, $K_\gamma(1318)$, $\eta_\alpha(775)$ are particles to be watched for.

Finally, a theory of the weak interactions has been considered. We assume that the weak decays of strongly interacting particles are induced by the weak decays of the aces which comprise them. From this followed :

i) the conserved vector current theory;

ii) $|\Delta I| = 1/2$, $\Delta S/\Delta Q = +1$ for $|\Delta S| = 1$ leptonic decays;

iii) $\Omega^-_\delta \to \Xi^0 + e^- + \nu$ is forbidden but $\Omega^-_\delta \to \Xi^0_\delta + e^- + \nu$ is allowed.

Numerical results for hyperon $\beta$ - decay have been presented.

There are, however, many unanswered questions. Are aces particles ? If so, what are their interactions ? Do aces bind to form only deuces and treys ? What is the particle (or particles) that is responsible for binding the aces ? Why must one work with masses for the baryons and mass squares for the mesons ? And more generally, why does so simple a model yield such a good approximation to nature ?

Our results may be viewed in several different ways. We might say :

1) the relationships we have established are accidents and our model is completely wrong. The formula $m(\Xi) = (3m(\Sigma)-m(N))/2$ is correct to electromagnetic mass splittings and yet seems entirely "accidental". It certainly would be no great surprise if our mass formulae were accidents too.

2) there is a certain simplicity present, additional to that supplied by the Eightfold Way, but this simplicity has nothing to do with our model [47]. For example, the Gell-Mann – Okubo mass formula may be written for any $SU_3$ representation as :

$$m^2 = m_o^2 \left\{1+b'(m_o^2)\left[I(I+1)-1/4\,Y^2\right]\right\}$$

for mesons,

$$m = m_o \left\{1+a(m_o)Y+b(m_o)\left[I(I+1)-1/4\,Y^2\right]\right\}$$

for baryons, where $m_o$, $b'$, $a$, $b$ vary from one representation to another. The quantities $b'$, $a$, and $b$ may be considered functions of $m_o$ or $m_o^2$. Equation (5.5) may be "explained" by postulating that $b'(m_o^2)$ goes like $b'(m_o^2) \sim 1/m_o^2$. Equation (3.15) would follow if $a(m_o)$ and $b(m_o)$ were any slowly varying functions of $m_o$, going for instance like $1/m_o$. Relations of this type could undoubtedly result from many different theories.

3) perhaps the model is valid inasmuch as it supplies a crude qualitative understanding of certain features pertaining to mesons and baryons. In a sense, it could be a rather elaborate mnemonic device.

4) there is also the outside chance that the model is a closer approximation to nature than we may think, and that fractionally charged aces abound within us.

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## Appendix A

The masses of the members of the baryon octet are:

$$938.211 \pm 0.01 = m(p) = 2m(1) + m(2) - 1/2\,(E_{11.} + E_{12.} + E_{1.1} + E_{1.2} + 2E_{.12})$$

$$939.505 \pm 0.01 = m(n) = m(1) + 2m(2) - 1/2\,(E_{22.} + E_{12.} + E_{2.2} + E_{1.2} + 2E_{.12})$$

$$1115.38 \pm 0.10 = m(\Lambda) = m(1) + m(2) + m(3) - 1/12\,(2E_{12.} + 5E_{13.} + 5E_{23.} + 2E_{1.2} + 5E_{1.3} + 5E_{2.3} + 8E_{.12} + 2E_{.13} + 2E_{.23})$$

$$1189.35 \pm 0.15 = m(\Sigma^+) = 2m(1) + m(3) - 1/2\,(E_{11.} + E_{13.} + E_{1.1} + E_{1.3} + 2E_{.13})$$

$$1193.2 \pm 0.7 = m(\Sigma^0) = m(1) + m(2) + m(3) - 1/4\,(2E_{12.} + E_{13.} + E_{23.} + 2E_{1.2} + E_{1.3} + E_{2.3} + 2E_{.13} + 2E_{.23})$$

$$1197.6 \pm 0.5 = m(\Sigma^-) = 2m(2) + m(3) - 1/2\,(E_{22.} + E_{23.} + E_{2.2} + E_{2.3} + 2E_{.23})$$

$$1315.2 \pm 1.0 = m(\Xi^0) = m(1) + 2m(3) - 1/2\,(E_{13.} + E_{33.} + E_{1.3} + E_{3.3} + 2E_{.13})$$

$$1321.2 \pm 0.3 = m(\Xi^-) = m(2) + 2m(3) - 1/2\,(E_{23.} + E_{33.} + E_{2.3} + E_{3.3} + 2E_{.23})$$

where we have suppressed the superscript 8 on all the binding energies.

The masses of the members of the baryon decuplet are:

$$
\begin{array}{llll}
1238 \pm & = m(\Delta^{++}_\delta) = 3m(1) & & - \quad (E_{11.} + \ldots) \\
& = m(\Delta^{+}_\delta) = 2m(1) + m(2) & & - 1/3\,(E_{11.} + 2E_{12.} + \ldots) \\
& = m(\Delta^{0}_\delta) = m(1) + 2m(2) & & - 1/3\,(E_{22.} + 2E_{12.} + \ldots) \\
& = m(\Delta^{-}_\delta) = 3m(2) & & - \quad (E_{22.} + \ldots) \\
1375 \pm 4 & = m(\Sigma^{+}_\delta) = 2m(1) & + m(3) & - 1/3\,(E_{11.} + 2E_{13.} + \ldots) \\
& = m(\Sigma^{0}_\delta) = m(1) + m(2) & + m(3) & - 1/3\,(E_{12.} + E_{13.} + E_{23.} + \ldots) \\
1392 \pm 4 & = m(\Sigma^{-}_\delta) = 2m(2) & + m(3) & - 1/3\,(E_{22.} + 2E_{23.} + \ldots) \\
& = m(\Xi^{0}_\delta) = m(1) & + 2m(3) & - 1/3\,(E_{33.} + 2E_{13.} + \ldots) \\
1533 \pm 3 & = m(\Xi^{-}_\delta) = m(2) & + 2m(3) & - 1/3\,(E_{33.} + 2E_{23.} + \ldots) \\
1686 \pm 12 & = m(\Omega^{-}_\delta) = & 3m(3) & - \quad (E_{33.} + \ldots)
\end{array}
$$

where we have suppressed the superscript 10 on all the binding energies. The expression $(E_{11.} + \ldots)$ stands for $(E_{11.} + E_{1.1} + E_{.11})$, with similar meanings for the other nine cases. The values and errors quoted for $m(\Sigma^{+}_\delta)$ and $m(\Sigma^{-}_\delta)$ are to be used to determine roughly the $\Sigma^{-}_\delta \Sigma^{+}_\delta$ mass difference. There may be systematic effects that equally shift both these masses. See reference [16].

## Appendix B

The vector - vector - pseudoscalar meson couplings are given by:

$$
\begin{aligned}
H_{int} \propto\ & \overline{\varphi}\,(K^{*-}K^{+} + K^{*+}K^{-} + \overline{K}^{*0}K^{0} + K^{*0}\overline{K}^{0} - 4/\sqrt{6}\,\varphi\eta) + \\
& \overline{K}^{*+}(K^{*0}\pi^{+} + 1/\sqrt{2}\,K^{*+}\pi^{0} + 1/\sqrt{2}\,\rho^{0}K^{+} + \rho^{+}K^{0} + 1/\sqrt{2}\,\omega K^{+} - 1/\sqrt{6}\,K^{*+}\eta + \\
& \qquad + (\varphi K^{+}) + \text{h.c.} + \\
& \overline{K}^{*0}(K^{*+}\pi^{-} - 1/\sqrt{2}\,K^{*0}\pi^{0} - 1/\sqrt{2}\,\rho^{0}K^{0} + \rho^{-}K^{+} + 1/\sqrt{2}\,\omega K^{0} - 1/\sqrt{6}\,K^{*0}\eta + \\
& \qquad + (\varphi K^{0}) + \text{h.c.} + \\
& \overline{\omega}\,(\sqrt{2}\,\rho^{-}\pi^{+} + \sqrt{2}\,\rho^{0}\pi^{0} + \sqrt{2}\,\rho^{+}\pi^{-} + \sqrt{2/3}\,\omega\eta + 1/\sqrt{2}\,K^{*-}K^{+} + 1/\sqrt{2}\,\overline{K}^{*0}K^{0} + \\
& \qquad + 1/\sqrt{2}\,K^{*0}\overline{K}^{0} + 1/\sqrt{2}\,K^{*+}K^{-}) + \\
& \overline{\rho}^{+}(\sqrt{2}\,\omega\pi^{+} + \sqrt{2/3}\,\rho^{+}\eta + K^{*+}\overline{K}^{0} + \overline{K}^{*0}K^{+}) + \text{h.c.} + \\
& \overline{\rho}^{0}(\sqrt{2}\,\omega\pi^{0} + \sqrt{2/3}\,\rho^{0}\eta + 1/\sqrt{2}\,K^{*-}K^{+} - 1/\sqrt{2}\,\overline{K}^{*0}K^{0} - 1/\sqrt{2}\,K^{*0}\overline{K}^{0} + \\
& \qquad + 1/\sqrt{2}\,\overline{K}^{*+}K^{-})
\end{aligned}
$$

The space-time part of the interaction has been suppressed.

The vector - pseudoscalar - pseudoscalar meson couplings are determined to be:

$$
\begin{aligned}
H_{int} \propto\ & \overline{\varphi}\,(K^-K^+ + \bar{K}^0K^0) \ + \\
& \overline{K^{*+}}\,(\pi^+K^0 + 1/\sqrt{2}\,\pi^0K^+ + \sqrt{3/2}\ \eta K^+) + h.c. \ + \\
& \overline{K^{*0}}\,(\pi^-K^+ + 1/\sqrt{2}\,K^0\pi^0 + \sqrt{3/2}\ \eta K^0) + h.c. \ + \\
& \overline{\omega}\,(1/\sqrt{2}\,K^+K^- + 1/\sqrt{2}\,K^0\bar{K}^0) \ + \\
& \overline{\rho^+}(\sqrt{2}\,\pi^0\pi^+ + K^+\bar{K}^0) + h.c. \ + \\
& \overline{\rho^0}(\sqrt{2}\,\pi^+\pi^- + 1/\sqrt{2}\,K^+K^- + 1/\sqrt{2}\,\bar{K}^0K^0)
\end{aligned}
$$

where the space-time part of the interaction is obtained by replacing the vector meson $V$ by $V_\mu$ and the product $P_1P_2$ of two pseudoscalar mesons by $(\partial_\mu P_1)P_2 - P_1(\partial_\mu P_2)$.

## Appendix C

We could consider an entirely different coupling scheme from that presented in Section VII where we do not restrict annihilations to take place between aces and anti-aces at corresponding triangle vertices, that is

$$T^{abc}T_{adb}D^{d}_{c} \tag{C.1}$$

or

$$T^{abc}T_{bad}D^{d}_{c} \tag{C.2}$$

might no longer be zero (in these examples we assume $c \neq a,b$; $d \neq a,b$; $a \neq b$; no summation over repeated indices). We call annihilations like the one given in (C.1), "1" type while that of (C.2) is "0" type. The 1 in "1" type indicates that aces annihilate at corresponding triangle vertices only once. When counting the number of annihilations at corresponding triangle vertices we do not include cases where the "dumbbell" or deuce helps. Hence (C.2) is "0" and not "1". We may also include the possibility of attaching minus signs to certain annihilation configurations. One natural way of proceeding is the following. We label the vertices of a triangle in clockwise order by 1, 2, 3. The action of a dumbbell on two triangles always picks out two vertices and hence two numbers. If these numbers are $a$ and $b$, then we may pick a $+$ or $-$ sign for the annihilation according to the value of $(-1)^{a+b}$. We indicate this coupling scheme by the symbol P (for permutation). Hence, (C.1) is $+1$ in "1" type coupling, $-1$ in "1P" type coupling, and 0 in "0" and "0P" type coupling. Clearly "2" = "2P" = the coupling type we have previously labelled in Section VII as $(+ + +)$. Counting all distinguishable annihilation configurations we obtain the couplings given in Appendix Table 1.

Only type (2 + 1)P generates couplings in all three representations. We do not work with (2 + 1)P coupling in this paper primarily because it gives a F/D ratio that is incompatible wih our speculations on the $\gamma$ - octet (Section X).

Our normalization has been such that:

a) S couplings are given by $(\overline{\Lambda}_\delta / \sqrt{3})\ (pK^- + n\overline{K}^0 + \ldots)$

b) F " " " " $\pi^+ (\overline{p}n - \overline{\Xi}^0\ \Xi^- + \ldots)$

c) D " " " " $\pi^+ (\overline{p}n + \overline{\Xi}^0\ \Xi^- + \ldots)$

d) T " " " " $\sqrt{2}\ \ \overline{\Delta^{++}_\delta} (p\pi^+ - \Sigma^+K^+)$

The detailed couplings may be found in reference [36].

Appendix Table 1

| Type of Coupling for Baryon → Baryon + Meson | Representation of Decaying Baryon | | |
|---|---|---|---|
| | Singlet | Octet | Decuplet |
| 2 = 2P = (+ + +) | 0 | F | 0 |
| 2 + 1 | 0 | 3(F + D)/2 | 0 |
| (2 + 1)P | -S | 3(F - D/3)/2 | T |
| 2 + 1 + 0 | 0 | 0 | 0 |
| (2 + 1 + 0)P | 0 | F - 3D | 2T |

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# R E F E R E N C E S


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4) S. Sakata, Prog. Theor. Phys. 16, 686 (1956).

5) Dr. Gell-Mann has independently speculated about the possible existence of these particles. His primary motivation for introducing them differs from ours in many respects. See Physics Letters 8, 214, 1964.

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

7) R. E. Behrends, J. Dreitlein, C. Fronsdal and W. Lee, Rev. Mod. Phys., 34, 1 (1962).

8) S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

9) Note that we use the identity $T_{ab,c} + T_{bc,a} + T_{ca,b} = 0$ .

10) For example, $E^8_{ab}$. is the binding energy between the aces a and b when they are positioned as in Figure 1a.

11) Since $A_1$ and $A_2$ form an isospin doublet their mass difference must be electromagnetic in origin and hence negligible in first approximation.

12) These formulae are obtained by counting the number of shaded squares (the number of $A_3$'s) that are present in each baryon. We have averaged the masses of the $\Lambda$ and $\Sigma$ somewhat arbitrarily in Eq. (3.3).

13) $E^8_{31.} = E^8_{32.}$ and $E^8_{3.1} = E^8_{3.2}$ if we neglect the electromagnetic interactions which distinguish ace 1 and 2 .

14) This formula was first derived with other techniques by: S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

15) This formula has been derived with other methods by S. Okubo, Physics Letters 4, 14 (1963).

16) This rather crude estimate of the $\Sigma^{-}_{\delta}\Sigma^{+}_{\delta}$ mass difference comes from W. A. Cooper, et al., to be published in "Physics Letters".

17) R. J. Oakes, Phys. Rev. 132, 2349 (1963).

18) S. P. Rosen, Phys. Rev. Letters 11, 100 (1963).

19) For example, we would write $m^2(\rho^+) = m_1^2 + m_2^2 - (E_1^2)^2$ where $E_1^2$ is the binding between ace 1 and anti-ace 2 . The author has no explanation for why squares of masses or binding energies should appear when working with mesons. This is especially mysterious in any model, like ours, where particles are treated as composite.

20) J. J. Sakurai, Phys. Rev. 132, 434 (1963).

21) For a discussion of the G matrix see: S. Okubo, Physics Letters 5, 165 (1963).

22) The existence of the $\pi^o_o$ with a mass near that of the pion is ruled out experimentally. For example, in addition to the decay $K^+ \rightarrow \pi^+ + \pi^o$ we would also expect to have $K^+ \rightarrow \pi^+ + \pi^o_o$ if $\pi^o_o$ existed. In fact, since the former decay mode is suppressed by the $|\Delta I| = 1/2$ rule while the latter is not, $K^+ \rightarrow \pi^+ + \pi^o_o$ would be the dominant $K^+$ decay mode.

23) If we identify the physical $\eta$ (550) with the $\pi^o_o$ we would expect the ninth pseudoscalar meson to be at $\sim$ 1300 MeV (we use the analogue of (4.12)). In this scheme, (5.4) would have to be viewed as an accident.

24) If the amplitude for $\varphi \to \rho\pi$ is given by: $A = \gamma_{\varphi\rho\pi}\,\varepsilon_{abcd}\,\varepsilon^{\varphi}_{a}\,p^{\varphi}_{b}\,\varepsilon^{\rho}_{c}\,p^{\rho}_{d}$ where $\varepsilon^{\varphi}$, $p^{\varphi}$, $\varepsilon^{\rho}$, $p^{\rho}$ are polarization and momentum four-vectors, then $\Gamma(\varphi \to \rho\pi) = \gamma^2_{\varphi\rho\pi}\,(p^{\rho})^3/36\pi$.

25) P. L. Connolly, et al., Phys. Rev. Letters 10, 371 (1963).

26) N. Gelfand, et al., Phys. Rev. Letters 11, 438 (1963).

27) R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

28) Dr. Gell-Mann discusses a similar theory from a different viewpoint. M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

29) N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

30) A. Halsteinslid, et al., Sienna International Conference on Elementary Particles, Sept. 30, 1963.

31) S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

32) M. Taher- Zadeh, et al., Phys. Rev. Letters 11, 470 (1963).

33) M. H. Alston, et al., "1962 International Conference on High Energy Physics at CERN", J. Prentki editor, pg. 311; G. Alexander, et al., Phys. Rev. Letters 8, 447 (1962).

34) P. L. Bastien and J. P. Berge, Phys. Rev. Letters 10, 188 (1963).

35) L. W. Alvarez, et al., Phys. Rev. Letters 10, 184 (1963).

36) S. Glashow and J. J. Sakurai, Nuovo Cimento 25, 337 (1962).

37) Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963).

38) V. Hagopian and W. Selove, Phys. Rev. Letters 10, 533 (1963); Z. Guiragossian, Phys. Rev. Letters 11, 85 (1963). These articles give references to earlier works.

39) S. Goldhaber, UCRL Report 10955 (1963).

40) M. Abolins, et al., Phys. Rev. Letters 11, 381 (1963).

41) R. Armenteros, et al., Sienna International Conference on Elementary Particles, Sept. 30, 1963.

42) D. D. Carmony, et al., preprint Jan. 20, 1964.

43) W. F. Frazer, S. H. Patil, N. Xuong, Phys. Rev. Letters 12, 178 (1963).

44) $m_o = 1350$ corresponds to the $\gamma$-octet + $\gamma$-singlet case. If there is no $\gamma$ - singlet we must use $m_o = 1410$.

45) G. Goldhaber, et al., UCRL Report 11183 (1964).

46) R. Armenteros, private communication.

47) In a recent preprint S. Coleman and S. L. Glashow have considered another mass formulae producing model.

# TABLE CAPTIONS

**Table 1** Here I, S, B, Q, J, and P stand for isospin, strangeness, baryon number, charge, spin, and parity, respectively. The lower limit on the ace mass is obtained by requiring that it be at least 1/3 the mass of the $\delta$ - decuplet.

**Table 4** $M_{\gamma,\delta}$, p, and $\Gamma$ represent the mass of the decaying baryon, the final state momentum, and the width. Decay modes that have not yet been observed are included within parentheses in column 1. Although the $\pi\Xi$ decay channel of the $\Xi_{\gamma}$ is suppressed by unitary symmetry, the large phase space available for this mode coupled with the breaking of the symmetry may account for the fact that $\Xi_{\gamma} \rightarrow \pi\,\Xi$ has been seen.

**Table 6** $M_{\gamma,\delta}$, p, $\sum|A|^2$, and $\Gamma$ are the mass of the decaying meson, the momentum in the final state, the sum over all charge states of the square of the decay amplitude, and the width for decay, respectively. The subscripts $\gamma$, $\delta$ indicate that the decaying meson has $J^P = 1^-$ or $1^+$.

**Table 7** We list here the low angular momentum systems that may be formed from an ace and an anti-ace. Certain resonances have been tentatively classified in this scheme. $\langle \bar{S} \cdot \bar{L} \rangle$ gives the expected value of the spin times the orbital angular momentum. It is tempting to conjecture that this is a pertinent quantity in ordering the energy levels of the $\bar{A}\,A$ system.

| Ace | I | $I_Z$ | S | B | Q | J | P | | Δ Mass (MeV) | Lifetime | Types of Coupling |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p_o(A_1)$ | 1/2 | +1/2 | 0 | 1/3 | 2/3 | 1/2 | + | >400 | 2 | $\sim\infty$ | a) "Strong" or perhaps very much stronger than "strong". |
| $n_o(A_2)$ | 1/2 | −1/2 | 0 | 1/3 | −1/3 | 1/2 | + | >400 | 150 | ~ minutes | |
| $\Lambda_o(A_3)$ | 0 | 0 | −1 | 1/3 | −1/3 | 1/2 | + | >550 | | $\sim 10^{-10}$ sec | b) Electromagnetic. |
| $\bar{p}_o(\bar{A}_1)$ | 1/2 | −1/2 | 0 | −1/3 | −2/3 | 1/2 | − | >400 | 2 | $\sim\infty$ | c) Weak. |
| $\bar{n}_o(\bar{A}_2)$ | 1/2 | +1/2 | 0 | −1/3 | +1/3 | 1/2 | − | >400 | 150 | ~ minutes | d) Gravitational. |
| $\bar{\Lambda}_o(\bar{A}_3)$ | 0 | 0 | +1 | −1/3 | +1/3 | 1/2 | − | >550 | | $\sim 10^{-10}$ sec | |

**TABLE 1**

Ace Properties

| Type of Coupling for Baryon → Baryon + Meson | Representation of Decaying Baryon | | |
| --- | --- | --- | --- |
| | Singlet | Octet | Decuplet |
| (+ + +) | 0 | F | 0 |
| (+ - +) | S | (F+D)/2 | T |
| (+ + -) | S | (F+D)/2 | -T |
| (+ - -) | 2S | D | 0 |

TABLE 2

| Decay | Cabibbo Theory | | Ace Theory | | Experiment | |
|---|---|---|---|---|---|---|
| | Interaction | Branching Ratio (units of $10^{-3}$) | Interaction | Branching Ratio (units of $10^{-3}$) | Interaction | Branching Ratio (units of $10^{-3}$) |
| $\Lambda \to p\, e^- \nu$ | V-0.72 A | 0.75 | V-A | 0.85 (input) | $V-1.0^{-0.7}_{+0.3}$ A | 0.85 ± 0.09 |
| $\Sigma^- \to n\, e^- \nu$ | V+0.65 A | 1.9 | V-A | 2.1 | | 1.3 ± 0.3 |
| $\Xi^- \to \Lambda\, e^- \nu$ | V+0.02 A | 0.35 | V-A | 1.2 | | 2.4 ± 1.4 |
| $\Xi^- \to \Sigma^0 e^- \nu$ | V-1.25 A | 0.07 | V-A | 0.042 | | |
| $\Xi^0 \to \Sigma^+ e^- \nu$ | V-1.25 A | 0.26 | V-A | 0.13 | | |
| $\Sigma^- \to \Lambda\, e^- \nu$ | | 0.09 (input) | 0(~¼A) | 0 (~0.01) | | 0.06 ± 0.03 |
| $\Omega_\delta^- \to \Xi^0 e^- \nu$ | ? | | forbidden | | | |
| $\Omega_\delta^- \to \Xi_\delta^0 e^- \nu$ | ? | | allowed | | | |

TABLE 3

Baryon $|\Delta S| = 1$ Leptonic Decays

| Decays of Octet | $M_Y$ (MeV) | p (MeV) | $\Gamma_{theory}$ (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|
| $N_Y \to \pi N$ | 1515 ± 3 | 452 | 80 (input) | 80 |
| $\Lambda_Y \to \bar{K}N$ | 1635? | 376 | 25 | < 40 |
| $(\pi\Sigma)$ | | 362 | 7.6 | |
| $\Sigma_Y \to \pi\Lambda$ | 1660 ± 10 | 441 | 6.6 | 13 |
| $(\bar{K}N)$ | | 402 | suppressed | < 2 |
| $\pi\Sigma$ | | 382 | 19.3 | 11 |
| $\Xi_Y \to \pi\Xi$ | 1770 ± 25 | 375 | suppressed | |
| $(\bar{K}\Lambda)$ | | 342 | 1.6 | |
| $(\bar{K}\Sigma)$ | | 246 | 2.9 | |

| Decays of Singlet | $M_\delta$ (MeV) | p (MeV) | $\Gamma_{theory}$ (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|
| $\Lambda_\delta \to \pi\Sigma$ | 1519 ± 2 | 261 | 9 (input) | 9 |
| $\bar{K}N$ | | 238 | 4.5 | 5 |

TABLE 4

Partial Widths of the $\gamma$-Octet and $\delta$-Singlet

| Decays of Octet | $M_\alpha$ (MeV) | p (MeV) | $\Gamma_{theory}$ (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|
| $N_\alpha \to \pi N$ | 1685 ± 5 | 570 | 80 (input) | 80 |
| $(\eta N)$ | | 385 | 0.6 | < 20 |
| $(K\Lambda)$ | | 227 | 0.06 | < 2 |
| $\Lambda_\alpha \to \bar{K}N$ | 1815 ± 35 | 539 | 42 | 70 |
| $(\pi\Sigma)$ | | 502 | 9.0 | < 40 |
| $(\eta\Lambda)$ | | 345 | 0.2 | < 1.3 |
| $\Sigma_\alpha \to (\pi\Lambda)$ | 1875? | 596 | 9.8 | |
| $(\bar{K}N)$ | | 586 | suppressed | |
| $(\pi\Sigma)$ | | 545 | 32 | |
| $(\eta\Sigma)$ | | 332 | 0.2 | |
| $(K\Xi)$ | | 213 | 0.04 | |
| $\Xi_\alpha \to (\bar{K}\Lambda)$ | 1972? | 539 | 4.4 | |
| $(\pi\Xi)$ | | 533 | suppressed | |
| $(\bar{K}\Sigma)$ | | 478 | 17 | |
| $(\eta\Xi)$ | | 298 | 0.3 | |

| Decays of Decuplet | $M_\delta$ (MeV) | p (MeV) | $\Gamma_{theory}$ (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|
| $\Delta_\delta \to \pi N$ | 1238 | 232 | 94 (input) | 94 |
| $\Sigma_\delta \to \pi\Lambda$ | 1382 | 205 | 26 | 39 ± 7 |
| $\pi\Sigma$ | | 123 | 3.8 | < 1.5 |
| $\Xi_\delta \to \pi\Xi$ | 1533 | 153 | 9.0 | 7 ± 2 |
| $\Omega_\delta$ | 1686 | Decays weakly into $\pi\Xi, \bar{K}\Lambda, \bar{K}\Sigma?, \Xi_\delta e\nu, \Xi_\delta \mu\nu$ | | |

TABLE 5

Partial Widths of the $\alpha$ - Octet and $\delta$ - Decuplet

| Decays of Nonet | $M_{\gamma,\delta}$ (MeV) | p (MeV) | $\Sigma\|A\|^2$ | $\Gamma_\gamma$ (PP) (MeV) | $\Gamma_\gamma$ (VP) (MeV) | $\Gamma_\gamma$ ((2PP+VP)/3) (MeV) | $\Gamma_\delta$ (VP) (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|---|---|---|---|
| $\pi_{\gamma,\delta} \to$ $\pi\pi$ | 1220 | 594 | 2 | 90 | | 61 | | |
| $(K\bar{K})$ | | 355 | 1 | 10 | | 6 | | |
| $\omega\pi$ | | 339 | 2 | | 100 | 33 | 100 | |
| $(\varphi\pi)$ | | 135 | 0 | | 0 | 0 | 0 | |
| $\Gamma(\pi_{\gamma,\delta}$ total) | | | | 100 (input) | 100 (input) | 100 (input) | 100 (input) | 100 ± 20 |
| $\omega_{\gamma,\delta} \to$ $(K\bar{K})$ | 1220? | 355 | 1 | 10 | | 6 | | |
| $(\rho\pi)$ | | 365 | 6 | | 362 | 122 | 320 | |
| $\Gamma(\omega_{\gamma,\delta}$ total) | | | | 10 | 362 | 128 | 320 | |
| $K_{\gamma,\delta} \to$ $(K\pi)$ | 1320? | 557 | 3/2 | 48 | | 32 | | |
| $(K\eta)$ | | 401 | 3/2 | 18 | | 12 | | |
| $(K^*\pi)$ | | 341 | 3/2 | | 76 | 25 | 65 | |
| $(\rho K)$ | | 212 | 3/2 | | 18 | 6 | 40 | |
| $(\omega K)$ | | 155 | 1/2 | | 2 | 0.8 | 10 | |
| $\Gamma(K_{\gamma,\delta}$ total) | | | | 66 | 96 | 76 | 115 | |
| $\varphi_{\gamma,\delta} \to$ $(\bar{K}K)$ | 1410 | 501 | 2 | 41 | | 27 | | |
| $(\rho\pi)$ | | 493 | 0 | | 0 | 0 | 0 | |
| $(\omega\eta)$ | | 225 | 0 | | 0 | 0 | 0 | |
| $K^*\bar{K}+\bar{K}^*K$ | | 129 | 4 | | 11 | 4 | 57 | |
| $\Gamma(\varphi_{\gamma,\delta}$ total) | | | | 41 | 11 | 31 | 57 | $\lesssim$ 60 |

TABLE 6a

| Decays of Octet | $M_{\Upsilon,\delta}$ (MeV) | p (MeV) | $\Sigma\|A\|^2$ | $\Gamma_{\Upsilon}(PP)$ (MeV) | $\Gamma_{\Upsilon}(VP)$ (MeV) | $\Gamma_{\Upsilon}((2PP+VP)/3)$ (MeV) | $\Gamma_{\delta}(VP)$ (MeV) | $\Gamma_{exp.}$ (MeV) |
|---|---|---|---|---|---|---|---|---|
| $\pi_{\Upsilon,\delta} \to$ $\pi\pi$ | 1220 | 594 | 2 | 90 | | 61 | | |
| $(K\bar{K})$ | | 355 | 1 | 10 | | 6 | | |
| $\omega\pi$ | | 339 | 2 | | 100 | 33 | 100 | |
| $(\varphi\pi)$ | | 135 | 0 | | 0 | 0 | 0 | |
| $\Gamma(\pi_{\Upsilon,\delta}$ total) | | | | 100 (input) | 100 (input) | 100 (input) | 100 (input) | 100±20 |
| $K_{\Upsilon,\delta} \to$ $(K\pi)$ | 1365? | 583 | 3/2 | 51 | | 34 | | |
| $(K\eta)$ | | 437 | 3/2 | 22 | | 14 | | |
| $(K^*\pi)$ | | 376 | 3/2 | | 102 | 34 | 67 | |
| $(\rho K)$ | | 273 | 3/2 | | 39 | 13 | 48 | |
| $(\omega K)$ | | 230 | 1/2 | | 8 | 3 | 14 | |
| $\Gamma(K_{\Upsilon,\delta}$ total) | | | | 73 | 149 | 98 | 129 | |
| $\eta_{\Upsilon,\delta} \to$ $(\bar{K}K)$ | 1410 | 501 | 3 | 62 | | 41 | | |
| $(\rho\pi)$ | | 493 | 2 | | 308 | 103 | 109 | |
| $(\omega\eta)$ | | 225 | 2/9 | | 3 | 1 | 6 | |
| $K^*\bar{K}+\bar{K}^*K$ | | 129 | 2/3 | | 2 | 0.6 | 10 | |
| $\Gamma(\eta_{\Upsilon,\delta}$ total) | | | | 62 | 313 | 146 | 125 | $\lesssim$ 60 |

TABLE 6b

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| $\bar{A}A$ System | $\langle \vec{S}\cdot\vec{L} \rangle$ | $C_J^{PG}$ for $\pi$ like number | $m(\pi)$ | $m(K)$ | $m(\eta)$ or $m(\omega)$ $m(\varphi)$ |
|---|---|---|---|---|---|
| ${}^1S_0$ | $-3/8$ | ${}^{+}0^{-\,-}$ | 135 | 494 | 548 |
| ${}^3S_1$ | $5/8$ | ${}^{-}1^{-\,+}$ | 750 | 890 | 784 1019 |
| ${}^3P_0$ | $-11/8$ | ${}^{+}0^{+\,-}$ | 550? | 725 | 775? |
| ${}^1P_1$ | $-3/8$ | ${}^{-}1^{+\,+}$ | 1140? | 1232 [46] | 1260? or 1140? 1320? |
| ${}^3P_1$ | $-3/8$ | ${}^{+}1^{+\,-}$ | 1200 [45] | 1290? | 1320? or 1200 1370? |
| ${}^3P_2$ | $13/8$ | ${}^{+}2^{+\,-}$ | >1200? | $\sqrt{m^2(\pi)+0.22}$ | $\sqrt{m^2(\pi)+0.29}$ or $m(\pi)$ $\sqrt{m^2(\pi)+0.44}$ |
| ${}^3D_1$ | $-19/8$ | ${}^{-}1^{-\,+}$ | 1220 | 1320? | 1220? 1410 |
| ${}^1D_2$ | $-3/8$ | ${}^{+}2^{-\,-}$ | >1200? | $\sqrt{m^2(\pi)+0.22}$ | $\sqrt{m^2(\pi)+0.29}$ or $m(\pi)$ $\sqrt{m^2(\pi)+0.44}$ |
| ${}^3D_2$ | $-3/8$ | ${}^{-}2^{-\,+}$ | >1200? | " | " " " |
| ${}^3D_3$ | $21/8$ | ${}^{-}3^{-\,+}$ | >1200? | " | " " " |

TABLE 7

Possible Meson Representations

## FIGURE CAPTIONS

**Figure 1** These deuces and treys correspond to the units from which all known particles are constructed.

a. Members of the baryon octet are built from treys of this type. The shaded circles at the vertices are aces, while the solid lines denote binding energies. In the unitary symmetric limit the three aces a, b, and c are indistinguishable, as shown.

b. The decuplet baryons are formed from this type of trey. Octet and decuplet treys may have different ace bindings.

c. This trey is used to construct the unitary singlet.

d.,e. The deuces shown correspond to members of meson octets and singlets in the limit of unitary symmetry. The open circles are anti-aces.

**Figure 2** We view the baryon octet with unitary symmetry broken by the strong interactions. One of the three aces has now become distinguishable from the other two. It is pictured as a shaded square. Mass splittings are induced by making the squares heavier than the circles. Since the same set of aces are used to construct all particles, mass relations connecting mesons and baryons may be obtained.

**Figure 3** After $SU_3$ has been broken by the strong and electromagnetic interactions the baryon octet looks like this. The three aces are now completely distinguishable from one another. If we assume that $n_0$ (the triangle) is heavier than $p_0$ (the circle) and neglect shifts in binding energies due to the electromagnetic breaking of the symmetry we find the qualitatively correct result that within any charge multiplet, the more negative the particle, the heavier the mass.

**Figure 4** These are the members of the baryon decuplet after unitary symmetry has been broken by the strong interactions.

**Figure 5** The decuplet has been further resolved by the electromagnetic interactions.

**Figure 6** The unitary singlet $\Lambda_\beta$ is constructed from this combination of treys.

**Figure 7** The vector mesons are represented by these deuces. Note the $\omega$ and $\rho$ masses are the same while the $\varphi$ mass is twice the $K^*$ mass minus either the $\omega$ or $\rho$ mass.

**Figure 8** The isolated octet of pseudoscalar mesons is represented after $SU_3$ has been broken by the strong interactions and the $\pi^o_o$ has been removed.

**Figure 9** We display the pseudoscalar mesons after $SU_3$ has been broken by the strong and electromagnetic interactions.

**Figure 10** This is a computation of the coupling for the decay $\omega \rightarrow K^{*+} K^-$. First we let the deuce representing $K^{*+}$ act on $\bar{\omega}$ as shown in a. and b. Note that the $p_o$ (solid circle) of $K^{*+}$ annihilates the $\bar{p}_o$ (open circle) of $\bar{\omega}$. $K^-$ is now allowed to act, as indicated in c. The part of $\bar{\omega}$ that was partially annihilated by $K^{*+}$ is now completely destroyed by $K^-$, allowing vacuum to be projected onto vacuum. Note that the part of $\bar{\omega}$ which could not be broken down by $K^{*+}$ remains unscathed by $K^-$ and does not contribute to the coupling.

**Figure 11** We compute here the $\bar{\omega}\,\omega\,\eta$ coupling.

**Figure 12** The $\varphi\,\rho\,\pi$ coupling is zero, as indicated.

**Figure 13** a. The coupling of $T^{121}T_{321}D^3_1$.

b. The coupling of $T^{211}T_{231}D^3_1$.

Both a. and b. are +1 in F type coupling. In F + D coupling a. = 1 , b. = -1 , i.e. the position of the dumb-bell determines the sign of the coupling.

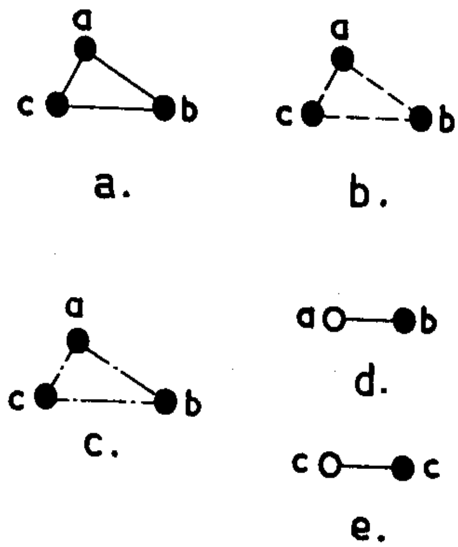



FIG. 1

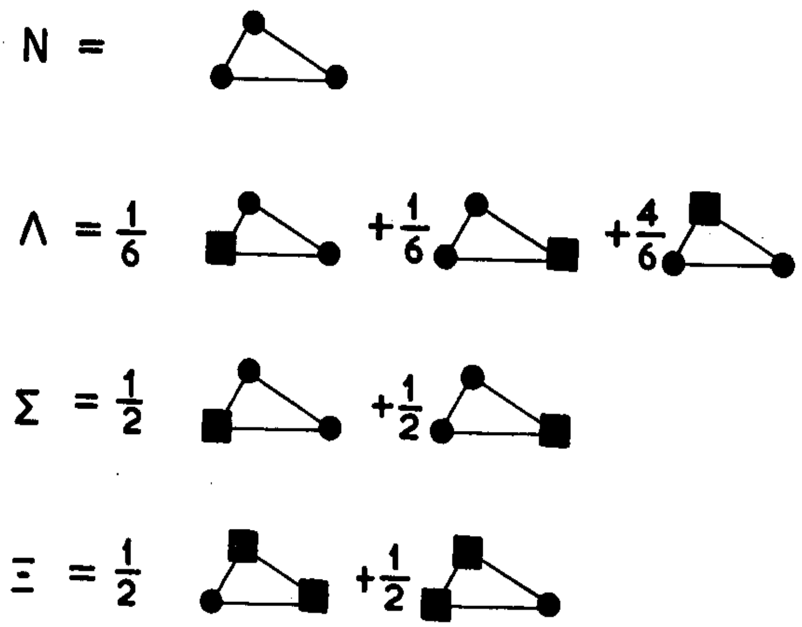



FIG.2

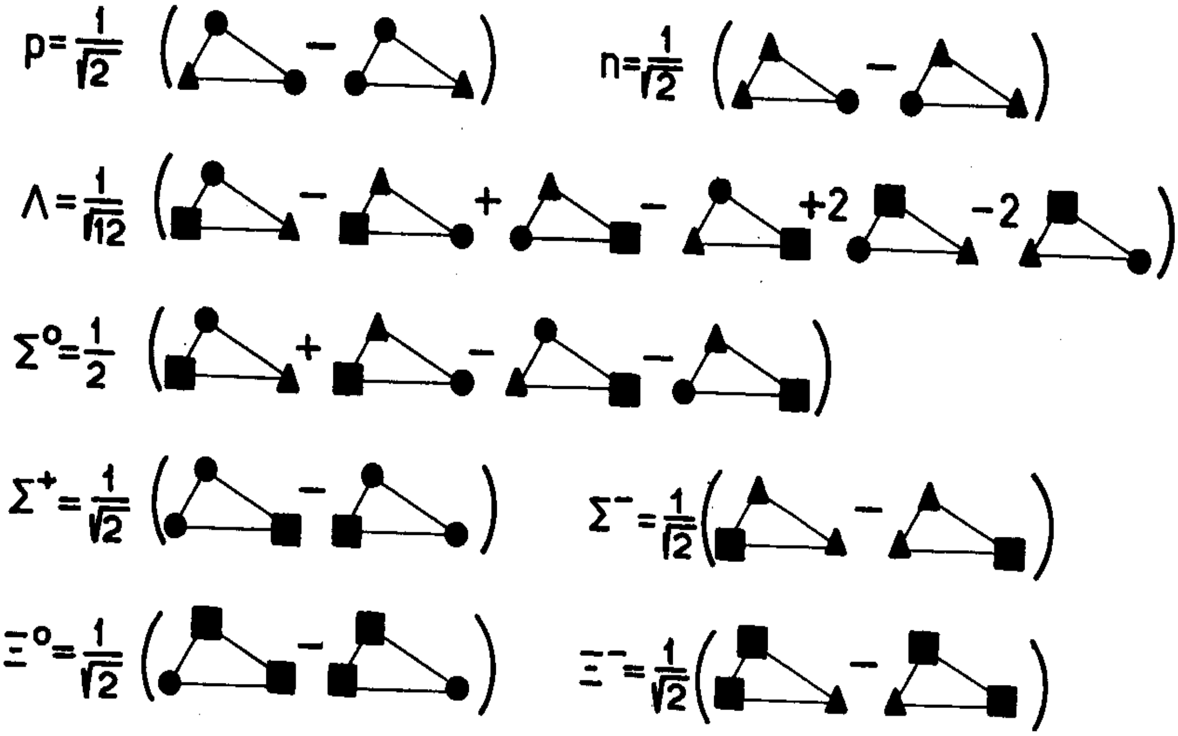



FIG. 3

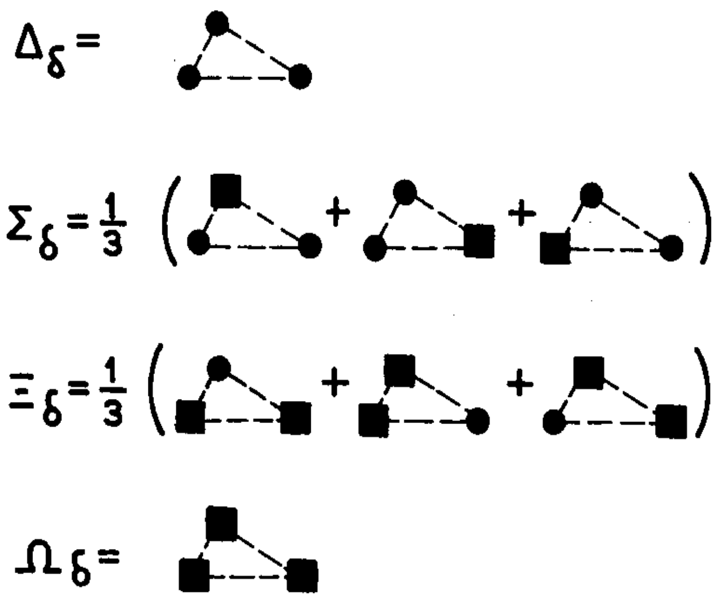



FIG. 4

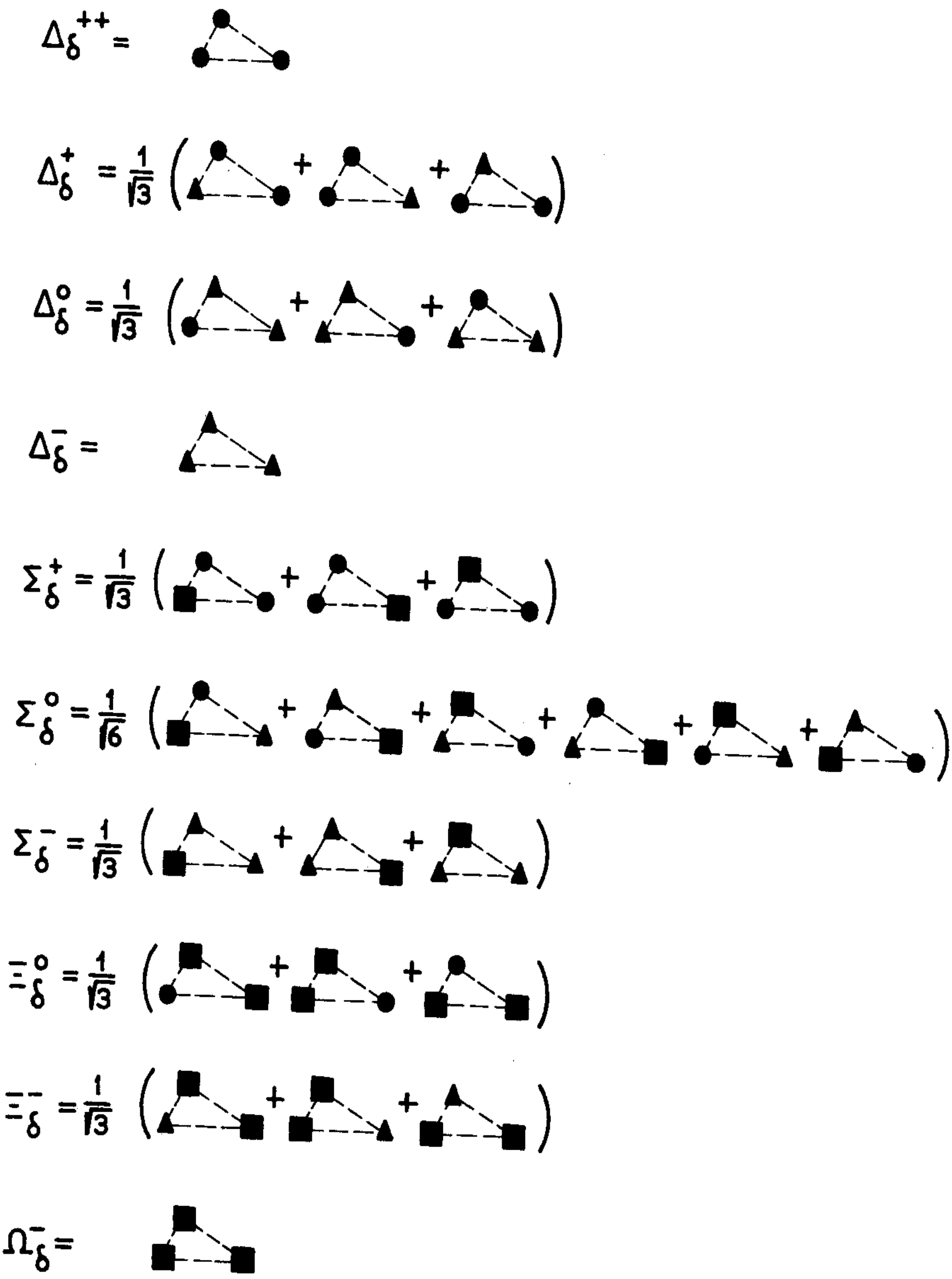

$\Delta_\delta^{++} =$
$\Delta_\delta^{+} = \frac{1}{\sqrt{3}}$ ( + + )
$\Delta_\delta^{o} = \frac{1}{\sqrt{3}}$ ( + + )
$\Delta_\delta^{-} =$
$\Sigma_\delta^{+} = \frac{1}{\sqrt{3}}$ ( + + )
$\Sigma_\delta^{o} = \frac{1}{\sqrt{6}}$ ( + + + + + )
$\Sigma_\delta^{-} = \frac{1}{\sqrt{3}}$ ( + + )
$\Xi_\delta^{o} = \frac{1}{\sqrt{3}}$ ( + + )
$\Xi_\delta^{-} = \frac{1}{\sqrt{3}}$ ( + + )
$\Omega_\delta^{-} =$

FIG. 5


FIG. 6

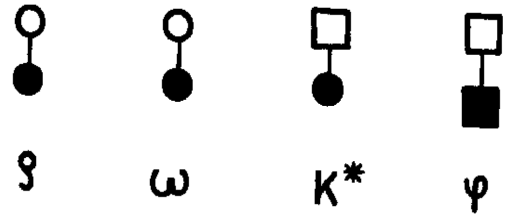



FIG. 7

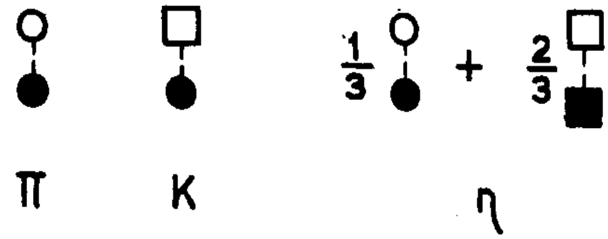



FIG. 8

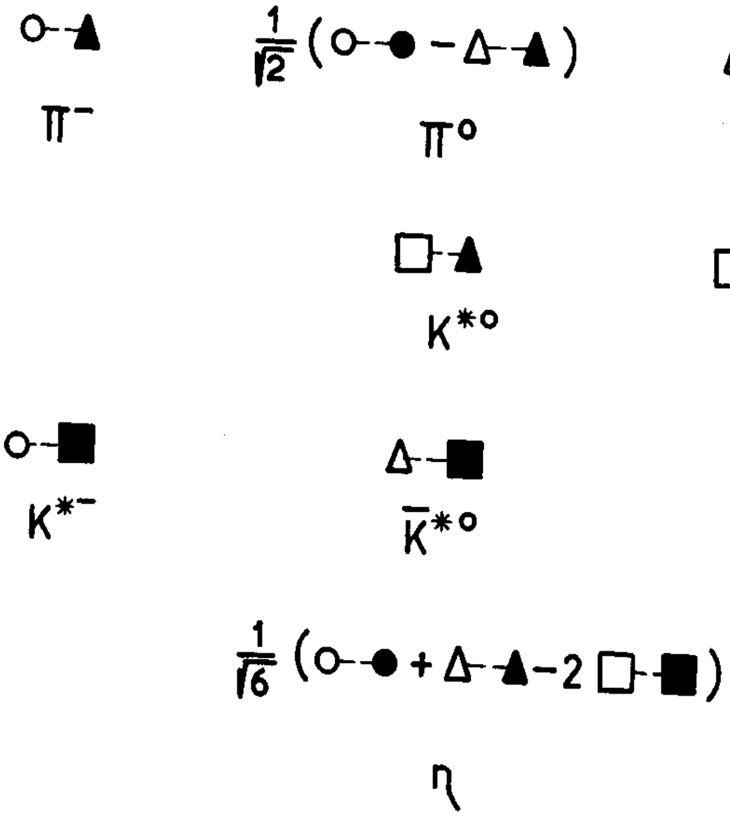



FIG. 9

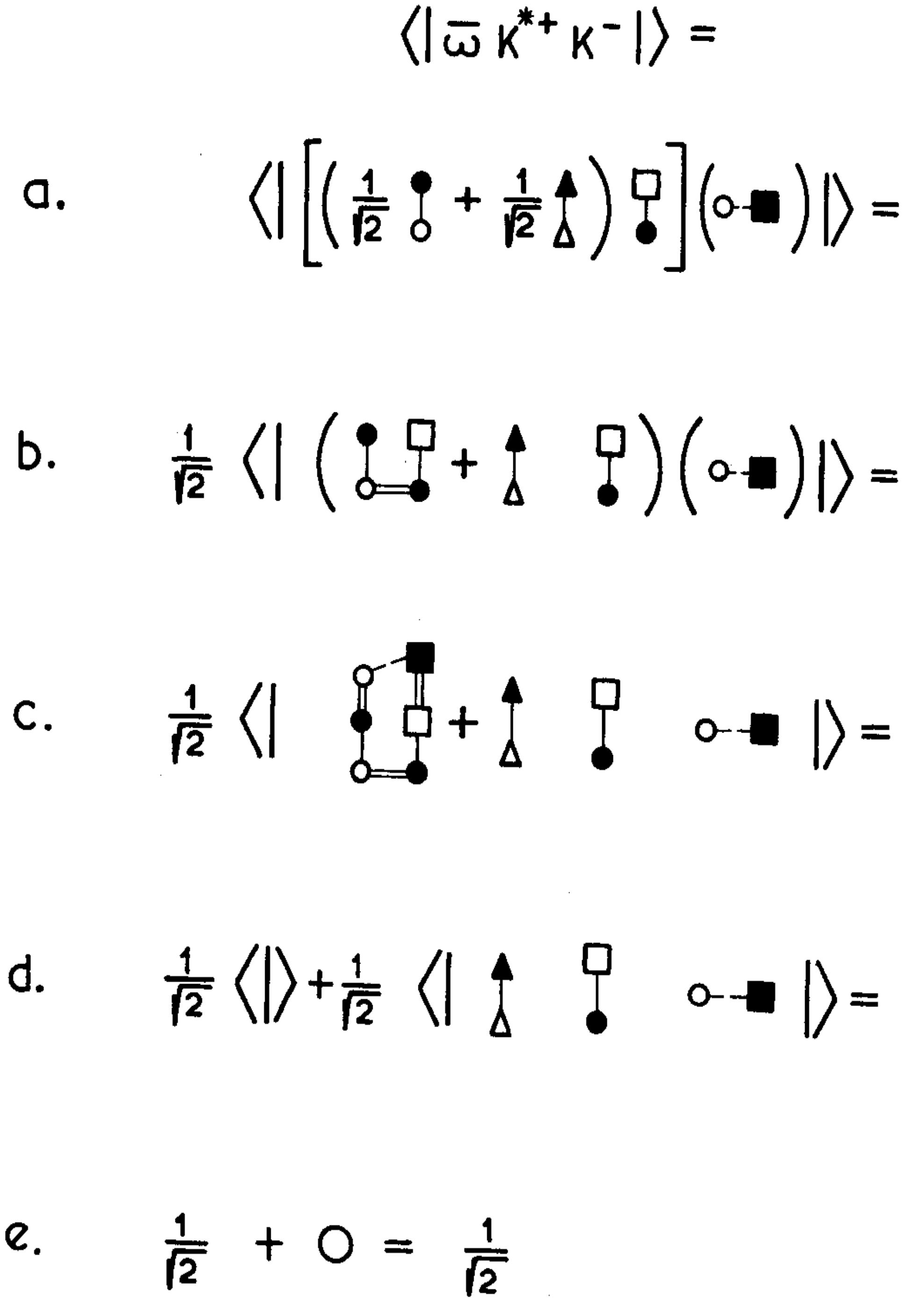



FIG. 10

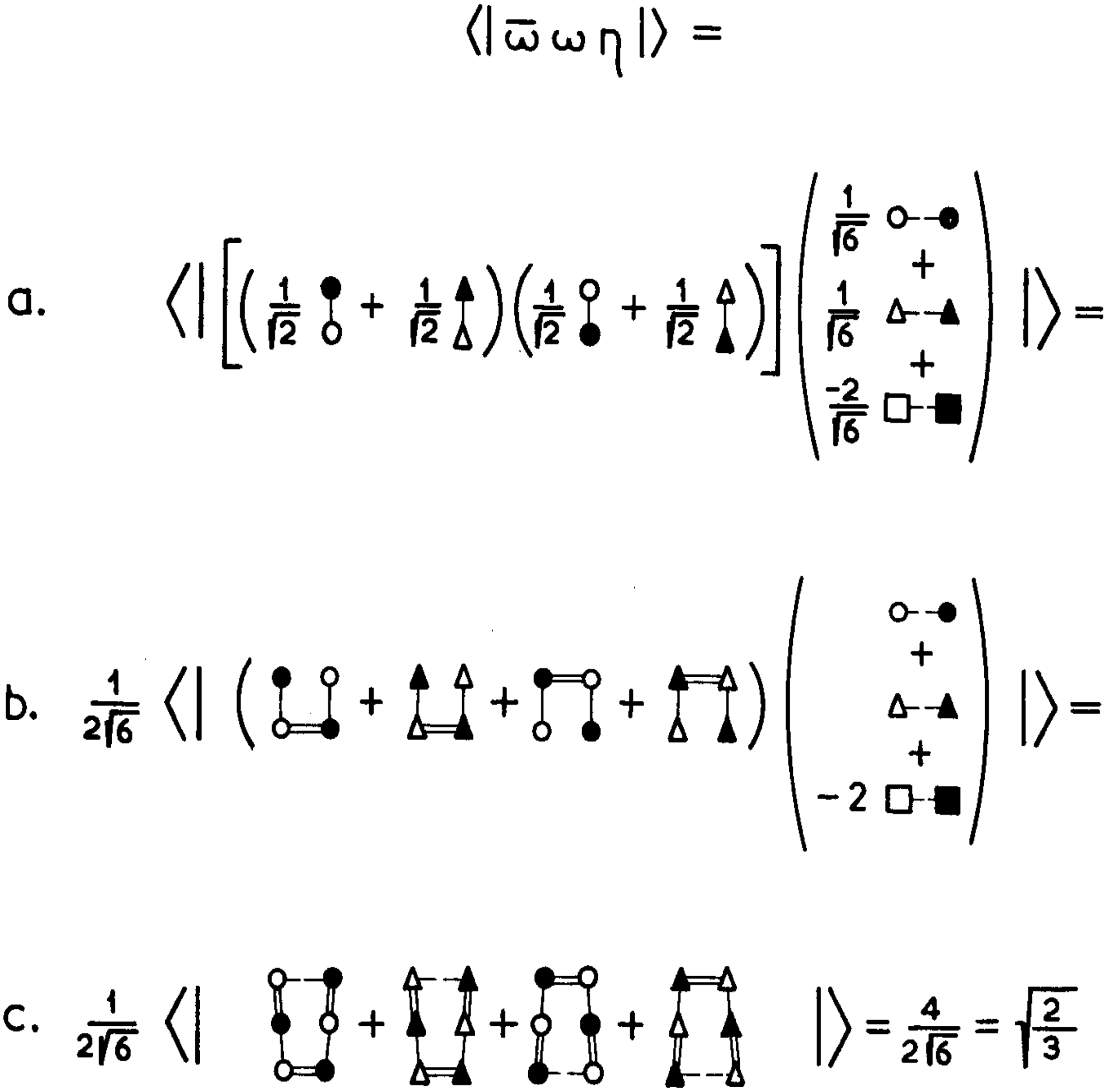



FIG. 11

$$\langle | \, \bar{\varphi} \, \rho^{+} \pi^{-} \, | \rangle =$$

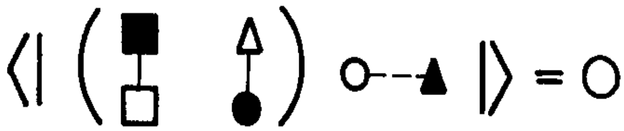
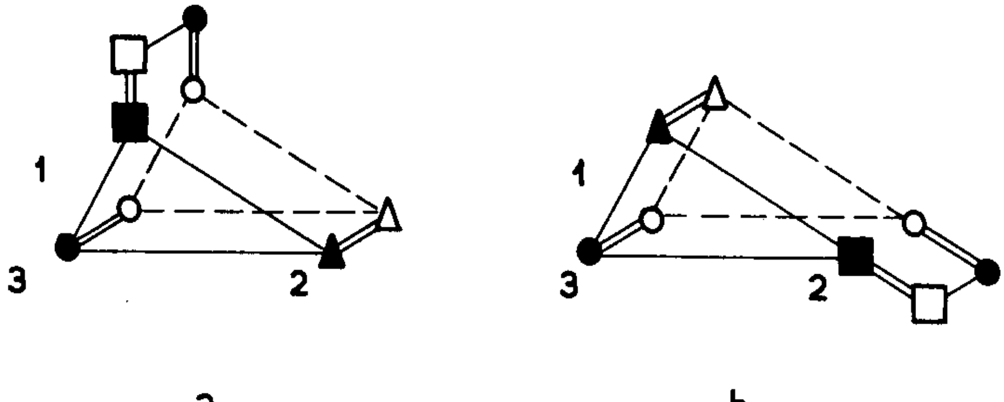


FIG. 12




a. b.